DISPENSA DI MECCANICA DEI FLUIDI

DI

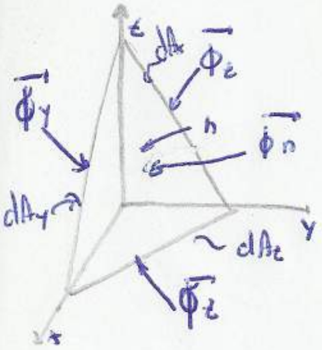
PIETRO ACETI

MECCANICA DEI FLUIDI

- STATI DI SFORZO

In un punto di fluido ∃ 3 velocità e ∞ stati di sforzo dipendenti dalla sup. considerata

Questi ∞ stati di sforzo possono essere ricondotti a solo 3 grazie al tetraedro di Cauchy



INFINITESIMI 2° ORDINE

INFINITESIMI 3° ORDINE

$$\sum F_{sup} + \sum F_{mas} = 0$$

$$\vec{\Phi}_n \, dA_n = \vec{\Phi}_x \, dA_x + \vec{\Phi}_y \, dA_y + \vec{\Phi}_z \, dA_z$$

$$\vec{\Phi}_n \, dA = \vec{\Phi}_x \, dA \cos \hat{nx} + \vec{\Phi}_y \, dA \cos \hat{ny} + \vec{\Phi}_z \, dA \cos \hat{nz}$$

$$\vec{\Phi}_n = \vec{\Phi}_x \cos \hat{nx} + \vec{\Phi}_y \cos \hat{ny} + \vec{\Phi}_z \cos \hat{nz}$$ VETTORE

$$\Phi_{nx} = \Phi_{xx} \cos \hat{nx} + \Phi_{yx} \cos \hat{ny} + \Phi_{zx} \cos \hat{nz}$$ ESEMPIO: COMPONE LUNGO X

Φ_{xy}

$$\rightarrow \underline{\underline{\Phi}} = \begin{bmatrix} \sigma_x & \tau_z & \tau_y \\ \tau_z & \sigma_y & \tau_x \\ \tau_y & \tau_x & \sigma_z \end{bmatrix}$$

• IDROSTATICA

Un fluido in quiete non ha spostamenti relativi tra le particelle $\Rightarrow \tau_x = \tau_y = \tau_z = 0$

∃ solo sforzi normali indipendenti dall'orientazione → $\sigma_x = \sigma_y = \sigma_z = p$

• PROPRIETÀ FISICHE

① DENSITÀ massa contenuta nell'unità di volume $\frac{Kg}{m^3} = \rho$

② PESO SPECIFICO: il peso dell'unità di volume $\frac{N}{m^3} = \frac{Kg\,a}{m^3} = \rho g = \gamma$

③ COMPRIMIBILITÀ

Se applico una differenza di pressione avrò una dif. di volume prop al V iniziale e a caret del flui

$$\boxed{dW = -\frac{W}{\varepsilon} dp} \quad (1)$$

Poiché la massa $m = cost$ — cons della massa

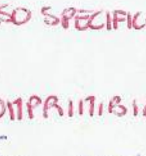
$$\rho W = cost \rightarrow d(\rho W) = 0 \Rightarrow \rho dW + W d\rho = 0 \Rightarrow \frac{-dW}{W} = \frac{d\rho}{\rho}$$

dalla ① $\rightarrow \frac{-dW}{W} = \frac{dp}{\varepsilon} \Rightarrow \frac{d\rho}{\rho} = \frac{dp}{\varepsilon} \Rightarrow$ INTEGRO TRA p e p_0, ρ e ρ_0 $\Rightarrow \ln\left(\frac{\rho}{\rho_0}\right) = \frac{p - p_0}{\varepsilon}$

$$\Rightarrow \rho = \rho_0 \, e^{\frac{(p - p_0)}{\varepsilon}}$$

$$\Rightarrow \boxed{\rho = \rho_0 \left[1 + \frac{p - p_0}{\varepsilon}\right]}$$

④ VISCOSITÀ



$$T = \mu A \frac{\Delta V}{\Delta r}$$

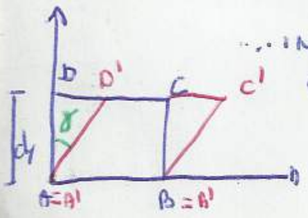
La forza resist T è proporzionale ① All'Area ② All'incremento di velocità che varia fra ③ alla distanza r ④ a un coef che dipende dal fluido

$$\downarrow \frac{1}{A}$$

$$\tau = \mu \frac{\Delta V}{\Delta r} \rightarrow \boxed{\tau = \mu \frac{\partial V}{\partial n}}$$ LEGGE DI NEWTON

N.B. QUANDO VARIA V ho le τ se $\Delta v = 0 \Rightarrow \tau = 0$ non ho sforzi τ in un fluido in quiete

⑤ FLUIDI NEWTONIANI



$V_A = V_B = V$; $V_C = V + \frac{dV}{dy} dy$

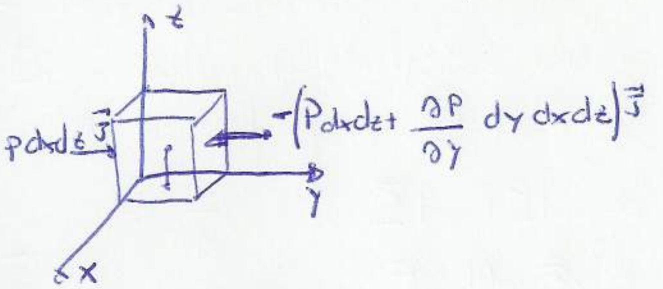
... IN UN ISTANTE dt ...

$$d\gamma = tg\, d\gamma = \frac{\frac{dV}{dy} \cancel{dy} \, dt}{\cancel{dy}} \Rightarrow \frac{d\gamma}{dt} = \frac{dV}{dy} \rightarrow \text{DALLA L. DI NEWTON} = \frac{d\gamma}{dt} = \frac{\tau}{\mu} \Rightarrow \boxed{\tau = \mu \frac{d\gamma}{dt}}$$

LA VELOCITÀ DI DEF ANGOLARE È PROP AL τ E DIPENDE DA μ

①

EQ INDEFINITA STATICA DEI FLUIDI



$$\sum F_m + \sum f_s = 0$$

$$\rho\vec{F}\,\cancel{dxdy\,dz} = \left(\frac{\partial P}{\partial x}\vec{i} + \frac{\partial P}{\partial y}\vec{j} + \frac{\partial P}{\partial z}\vec{k}\right)\cancel{dxdydz}$$

$$\boxed{\rho\vec{F} = \text{grad}(p)}$$ EQ INDEF DELLA STATICA DEI FLUIDI

$\vec{F}$ $F/_{Kg} = Acc = g$

FLUIDO PESANTE $(\vec{F} = \vec{g})$ INCOMPRIMIBILE $(\rho = cost)$

$-\vec{g}\,\text{grad}(z)$

VALE PER P e P*

↓

LEGGE DI STEVIN

$$\Rightarrow +\rho g\,\text{grad}(z) + \text{grad}(p) = 0 \Rightarrow \gamma\nabla z + \nabla p = 0 \quad \nabla z + \nabla\left(\frac{p}{\gamma}\right) = 0 \quad \nabla\left(z + \frac{p}{\gamma}\right) = 0 \Rightarrow \boxed{z + \frac{p}{\gamma} = cost}$$

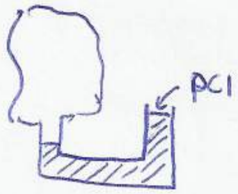
h geodetica, h di pressione, h piezometrica

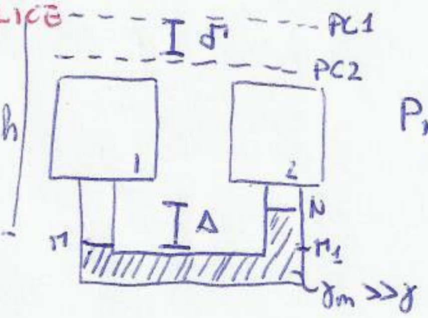
NB

Lungo le sup equipotenziali la pressione è la stessa. $p = (\overbrace{cost - z}^{cost})\gamma$

$F = \Delta U$ (Forze conservative) → $\rho\,\Delta U = \Delta p$ → su una sup equi $\Delta U = cost$ → $\Delta p = 0$ → $p = cost$

MANOMETRO SEMPLICE





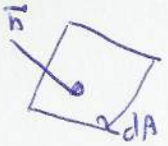
$$P_{M_1} = P_N + \Delta\gamma_m = \gamma(h - \Delta - \delta') + \Delta\gamma_m = \gamma h$$

$$-\Delta\gamma - \delta'\gamma + \Delta\gamma_m = 0$$

$$\boxed{\delta' = \frac{\Delta(\gamma_m - \gamma)}{\gamma}}$$

se $\gamma_m \leq \gamma \Rightarrow \delta' = \boxed{\frac{\Delta(\gamma - \gamma_m)}{\gamma}}$ se $\gamma_1 \neq \gamma_2$ $\boxed{\delta' = \frac{h(\gamma_2 - \gamma_1) + \Delta(\gamma_m - \gamma_2)}{\gamma_2}}$

SPINTE IDROSTATICHE



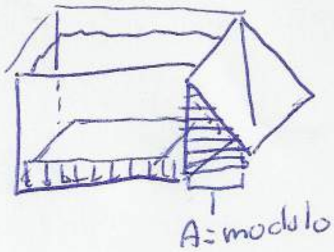
$$\text{Def} := d\bar{s} = p\, dA\, \bar{n} \rightarrow \boxed{\bar{S} = \int_A p\, \bar{n}\, dA}$$

DI UNA SPINTA BISOGNA CONOSCERE: ① MODULO ② DIR ③ VERSO ④ P.TO DI APPLICAZIONE

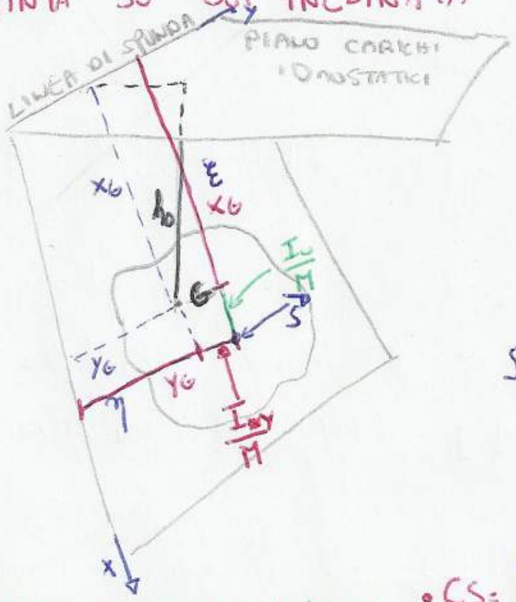
• SPINTE SUP. PIANE

sup piane $\bar{n} = \text{cost}$

- $|S| = \int_A p\, dA = pA$
- DIR ⊥ al fluido e uscente da esso
- BARICENTRO è il p.to di applicazione se la pressione è costante



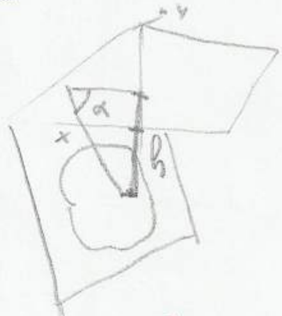
• SPINTA SU SUP INCLINATA



- SPINTA LA CALCOLO SULL'AREA
- IL centro invece la considero inclinata

$$\bullet\ |s| = \int_A p\, dA = \int_A \gamma\, x \sin\alpha\, dA = \gamma \sin\alpha \int_A x\, dA = \gamma \sin\alpha\, \underbrace{x_G A}_{h_G \to \text{AFFONDAMENTO BARICENTRO}}$$

$$|s| = \gamma\, \overset{\text{AFFONDAMENTO BARICENTRO}}{h_G}\, A = p_G A$$



$$\bullet\ S\xi = \int_A p\, dA_x = \int \gamma h\, dA_x =$$

$$S\xi = \int_A \gamma\, x \sin\alpha\, x\, dA = \gamma \sin\alpha \int_A x^2 dA$$

$$\xi = \frac{\gamma \sin\alpha \int_A x^2 dA}{S} = \frac{\gamma \sin\alpha \int_A x^2 dA}{\gamma \sin\alpha \int_A x\, dA} = \boxed{\xi = \frac{I}{M}}$$

$$\boxed{\eta = \frac{I_{xy}}{M} = \frac{\int xy\, dA}{\int x\, dA \sim x_G A}}$$

*2

$$\bullet\ CS = \Big(x_G + \frac{I_0}{\underset{x_G A}{M}} ;\ \underset{\text{simmetria}}{0} \Big)$$

x_G = AFFONDAMENTO DI P_G $\cdot \sin\alpha \cos\alpha$

$M = x_G A$

I_0 = DELLA FIGURA VISTA ⊥

• SUP CURVE

① EQUILIBRIO GLOBALE: RICONDUCO SUP CURVE ALLA SOMMA DI SUP PIANE

$$\begin{cases} \vec{\Pi}_1 + \vec{\Pi}_2 + \vec{G} = 0 \\ \bar{S} = -\vec{\Pi}_1 \end{cases}$$

$$-\vec{\Pi}_1 = +\underset{\rho W g}{\vec{G}} + \underset{\int_A p\, dA}{\vec{\Pi}_2}$$

② METODO DELLE COMPONENTI: SCOMPONGO LA SPINTA CURVA IN 3 SPINTE PIANE

$$dS_x = p\, dA_x \rightarrow S_x = \int_\pi p\, dA_x = \gamma h_{Gx} A_x$$

$$dS_y = p\, dA_y \rightarrow S_y = \int p\, dA_y = \gamma h_{Gy} A_y$$

$$dS_t = p\, dA_t \rightarrow S_t = \int p\, dA_t = \text{peso} = \gamma W$$

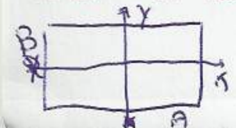
*1

$$X_1 = x_G + x'$$

$$\xi = \frac{I}{M} = \frac{\int x^2 A}{\int x A} = \frac{x_G^2 A + \int x'^2 dA}{\underset{S}{x_G A + x_1 A}} = x_G + \frac{I_0}{x_G}$$

I_0 momento di inerzia rispetto a un asse baricentrico // alla linea di sponda

MOMENTI DI



$$I_{xx} = \frac{AB^3}{12} \quad I_y = \frac{BA^3}{12}$$

CINEMATICA

LAGRANGE: SI SEGUE UNA PARTICELLA V VARIA nello spazio e nel tempo $\frac{d}{dt}$

EULERO: SI FISSA UN VOLUME E SI GUARDA COSA PASSA: V varia solo nel tempo $\frac{\partial}{\partial t}$

$$\vec V = \vec V(x, y, z, t)$$

$$\vec V = u\vec i + v\vec j + w\vec z \quad \text{con} \quad u = \frac{dx}{dt} \quad v = \frac{dy}{dt} \quad z = \frac{dz}{dt}$$

$$a = \frac{d\vec V}{dt} = \frac{d\vec V(x(t); y(t), z(t); t)}{dt} = \underbrace{\frac{\partial \vec V}{\partial t}}_{} + \overbrace{\left(\frac{\partial \vec V}{\partial x}u + \frac{\partial \vec V}{\partial y}v + \frac{\partial \vec V}{\partial z}w\right)}^{}$$

ACC CONV: come V varia con la posizione. non è lineare → origine turbolenze

$$\downarrow$$

$$\frac{\partial V}{\partial t} + \frac{\partial \vec V}{\partial x}\overbrace{\frac{dx}{dt}}^{u} + \frac{\partial \vec V}{\partial y}\overbrace{\frac{dy}{dt}}^{v} + \frac{d\vec V}{\partial z}\overbrace{\frac{dz}{dt}}^{w}$$

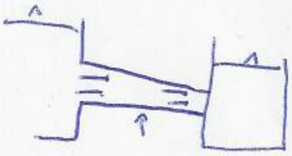
ACC LOCALE: come varia $\vec V$ solo rispetto al tempo
$\neq 0$ solo per moti non stazionari

LINEA DI CORRENTE: È UNA LINEA tg in ogni punto al vettore velocità del punto

TUBO DI FLUSSO = tra le linee di corrente posso delimitare una regione chiamata tubo di flusso dove non ho correnti ⊥ ad esso.

N.B. ~~[illegible]~~ UNA CONDOTTA è un tubo di flusso.

TIPI DI MOTO:



Permanente: $\vec V$ varia nello spazio ma non nel tempo

$$\vec V = \vec V(x, y, z)$$

Uniforme: Non varia

$$\vec V = cost$$

STAZIONARI

~~Stazionario~~

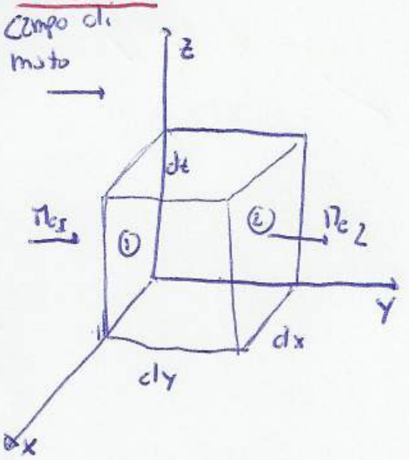
VARIO: il moto varia.

$$d\left[f(g(x))\right] = \frac{df}{dg}\frac{dg}{dx} \Rightarrow (3x)^2 = \frac{d(3x)^2}{d(3x)} \cdot \frac{d3x}{dx} \quad 2\,3x \cdot 3 =$$

$$a = \frac{d\vec V}{dt} \quad \frac{\partial V}{\partial t} + \overbrace{\frac{\partial V}{\partial x}\frac{dx}{dt}}$$

EQUAZIONI DI CONTINUITÀ : conservazione della massa

INDEFINITA



Nel volume considerato avrò $M = \rho\, dxdydz$

In ① $M_I = \rho \overbrace{v\, dxdz}^{Q}\, dt$

$$\frac{kg}{\cancel{m^3}}\frac{\cancel{m}}{\cancel{s}}\cancel{m}\cancel{m}\cancel{s} = kg$$

In ② $M_{II} = M_I + \frac{\partial M_{I}}{\partial y} dy$

ENTRANTE - USCENTE $= \cancel{M_I} - \cancel{M_I} - \frac{\partial M_I}{\partial y} dy = \overset{\perp y}{\overline{-\frac{\partial}{\partial y}(\rho v)\, dxdzdtdy}}$

$$\Delta M = -\frac{\partial}{\partial x}(\rho u)\, dxdydzdt - \frac{\partial}{\partial y}(\rho v)\, dxdydzdt - \frac{\partial}{\partial z}(\rho w)\, dxdydzdt$$

$$\frac{dm}{dt}dt = \frac{\partial(\rho\, dxdydz)}{\partial t}dt = \frac{\partial \rho}{\partial t}\, dxdydz\, dt$$

$$\Delta M = \frac{\partial \rho}{\partial t}\, dxdydz\, dt$$

VAR MASSA NEL VOLUME

L. $\boxed{\frac{\partial \rho}{\partial t} + \text{div}(\rho \vec{v}) = 0}$ FLUSSO SULLA SUP DI CONTORNO

parziale

DERIVATA COMPLETA DI ρ

RISCRIVO: $\frac{\partial \rho}{\partial t} + u\frac{\partial \rho}{\partial x} + \rho\frac{\partial v}{\partial y} + v\frac{\partial \rho}{\partial t} + \rho\frac{\partial u}{\partial x} + w\frac{\partial \rho}{\partial y} + \rho\frac{\partial w}{\partial z}$

$\boxed{\frac{d\rho}{dt} + \rho\, \text{div}(\vec{v}) = 0}$ — $\rho = \text{cost}$ FLUIDO INCOMPRIMIBILE $= \boxed{\text{div}(\vec{v}) = 0} \Rightarrow$ NON POSSO DIRE CHE È STAZIONARIO, MA POSSO DIRE CHE $\vec{v}$ è cost solo ~~in direzione~~ se il moto è monodirezionale

completa

GLOBALE → integro

$$\frac{\partial \rho}{\partial t} + \text{div}(\rho\vec{v}) = 0 = \int_W \frac{\partial \rho}{\partial t}\, dW + \int_W \text{div}(\rho\vec{v})\, dW = 0$$

TH STOKES

$$\int_W \text{div}(F)\, dW = \overset{\text{entrante}}{-}\int_A \vec{F}\cdot\vec{n}\, dA$$

$$= \underbrace{\frac{\partial}{\partial t}\int_W \rho\, dW}_{\text{come varia la m nel vol}} = \underbrace{\int_A \rho\vec{v}\cdot\vec{n}\, dA}_{\text{entrante meno uscente}} \Rightarrow \boxed{\frac{\partial}{\partial t}\int_W \rho\, dW = \int_A \rho\, v_n\, dA}$$

v_n — V in dir n

$\int_A v_n dA$ = PORTATA VOLUMETRICA $\frac{m}{s}m^2 = \frac{m^3}{s} = \dot{V}$

$$\Rightarrow \rho = \text{cos} \quad \overset{\to 0}{\frac{\partial m}{\partial t}} = \rho\int_A v_n\, dA \Rightarrow \int_A \rho\, v_n dA = 0 \Rightarrow \boxed{Q_E = Q_U}$$

TUBI DI FLUSSO

$M_{interno} = \rho A\, ds$

$M_{ENT} = \rho Q\, dt$

$M_{US} = \rho Q\, dt + \frac{\partial(\rho Q)}{\partial s} ds\, dt$

VARIAZIONE entrante ed esce — variazione massa

$$\Rightarrow -\frac{\partial(\rho Q)}{\partial s}\cancel{ds\, dt} = \frac{\partial(\rho A)}{\partial t}\cancel{ds\, dt}$$

$$\boxed{\frac{\partial(\rho A)}{\partial t} + \frac{\partial(\rho Q)}{\partial s} = 0}$$

LA PORTATA LUNGO S VARIA DALLA SEZIONE

FLUIDO INCOMPRIMIBILE $\rho = \text{cost}$

$\rho\frac{dA}{\partial t} + \rho\frac{dQ}{\partial s} = 0$ → LA PORTATA è costante quando $\frac{\partial A}{\partial t} = 0$ < MOTO PERMANENTE / CONDOTTA INDEFORMABILE

- PER UN MOTO PERMANENTE

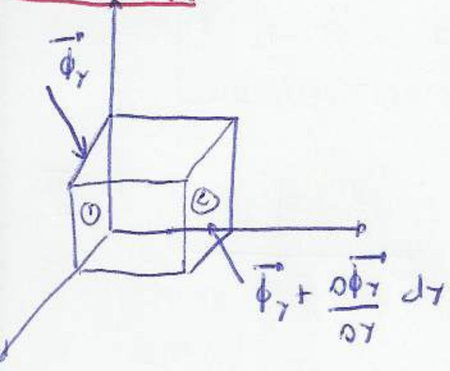
oppure

- PER UNA CONDOTTA INDEF.

→ $\boxed{Q_{Iniziale} = Q_{Finale}}$

BILANCIO QUANTITA DI MOTO : EQUILIBRIO DINAMICO

INDEFINITA



$$\sum \vec{F} = m\vec{a}$$

$$\sum \vec{F}_s + \sum \vec{F}_m = m\vec{a}$$

$$\sum \vec{F}_m = \rho \vec{f}\, dx\,dy\,dz$$

$$\sum \vec{F}_s \overset{\text{SOL}}{=}_{\text{LUNGO } y} = \left(\vec{\Phi}_y - \vec{\Phi}_y - \frac{\partial \vec{\Phi}_y}{\partial y} dy\right) dx\,dz \quad \text{ANALOGO PER } x, z$$

$$\rho \vec{f}\, dx\cancel{dy}dz - \frac{\partial \vec{\Phi}_x}{\partial x} dx\cancel{dy}dz - \frac{\partial \vec{\Phi}_y}{\partial y} dx\cancel{dy}dz - \frac{\partial \vec{\Phi}_z}{\partial z} dx\cancel{dy}dz = \rho\, dx\cancel{dy}dz\, \vec{a}$$

$$\boxed{\rho(\underset{\text{forzante}}{\vec{f}} - \underset{\text{termini cinematici}}{\vec{a}}) = \operatorname{div}(\underline{\Phi})}$$ distribuzione degli sforzi

$$\underline{\Phi} = \begin{bmatrix} \sigma_x & \tau_z & \tau_y \\ \tau_z & \sigma_y & \tau_x \\ \tau_y & \tau_x & \sigma_z \end{bmatrix}$$

GLOBALE

$$\underset{①}{\int_W \rho \vec{f}\, dW} - \underset{②}{\int_W \rho \vec{a}\, dW} = \underset{③}{\int_W \frac{\partial \vec{\Phi}_x}{\partial x} dW} + \underset{④}{\int_W \frac{\partial \vec{\Phi}_y}{\partial y} dW} + \underset{⑤}{\int_W \frac{\partial \vec{\Phi}_z}{\partial z} dW}$$

① $\vec{G} = m\vec{f}$

② $\rho \vec{a} = \rho \frac{d\vec{V}}{dt} = \rho \frac{\partial \vec{V}}{\partial t} + \overline{\rho u \frac{d\vec{V}}{dx}} + \overline{\rho v \frac{d\vec{V}}{dy}} + \overline{\rho w \frac{d\vec{V}}{dz}}$

$$= \underbrace{\rho \frac{d\vec{V}}{dt}} + \overline{\frac{\partial}{\partial x}(\rho u \vec{V}) - \vec{V} \underbrace{\frac{\partial(\rho u)}{\partial x}}} + \overline{\frac{\partial}{\partial y}(\rho v \vec{V}) - \vec{V} \underbrace{\frac{\partial(\rho v)}{\partial y}}} + \overline{\frac{\partial}{\partial z}(\rho w \vec{V}) - \vec{V} \underbrace{\frac{\partial(\rho w)}{\partial z}}}$$

$$= \rho \frac{d\vec{V}}{dt} + \frac{\partial}{\partial x}(\rho u \vec{V}) + \frac{\partial}{\partial y}(\rho v \vec{V}) + \frac{\partial}{\partial z}(\rho w \vec{V}) - \vec{V} \operatorname{div}(\rho \vec{V})$$

$$= \overline{\rho \frac{d\vec{V}}{dt}} + \quad '' \quad + \quad '' \quad + \quad '' \quad + \overline{\vec{V} \frac{\partial \rho}{\partial t}}$$

$$= \frac{\partial(\rho \vec{V})}{\partial t} + \frac{\partial}{\partial x}(\rho u \vec{V}) + \frac{\partial}{\partial y}(\rho v \vec{V}) + \frac{\partial}{\partial z}(\rho w \vec{V}) \Rightarrow \text{INTEGRANDO} \int_W \frac{\partial(\rho \vec{V})}{\partial t} dW + \int_W \frac{\partial(\rho u \vec{V})}{\partial t} \ \ldots\ldots$$

NOTO CHE:

$$\frac{\partial}{\partial x}(\rho u \vec{V}) = \overset{\text{SOST}}{\overline{\rho u \frac{\partial \vec{V}}{\partial x}}} + \vec{V} \frac{\partial(\rho u)}{\partial x}$$

EQ DI CONTINUITÀ:

$$\rho \operatorname{div}(\vec{V}) + \frac{\partial \rho}{\partial t} = 0 \Rightarrow \operatorname{div}(\rho \vec{V}) = -\frac{\partial \rho}{\partial t}$$

$$\Rightarrow \text{TH DI STOKES} = \int_W \frac{\partial(\rho \vec{V})}{\partial t} dW + \int_A \rho \vec{V} (u \cos \hat{n}_x + v \cos \hat{n}_y + z \cos \hat{n}_z) = \boxed{\frac{\partial}{\partial t} \int_W \frac{\rho \vec{V}}{\cancel{\partial t}} dW - \int_A \rho \vec{V} V_n}$$

③ $\int_W \frac{\partial \phi_x}{\partial x} dW = -\int_A \phi_x \cos(\hat{n}_x)\, dA$

$$\Rightarrow \vec{G} \underbrace{- \frac{\partial}{\partial t} \int_W \rho \vec{V}\, dW}_{\substack{\text{DERIVATA QUANTITÀ} \\ \text{DI MOTO: INERZIE}}} + \underbrace{\int_A \rho \vec{V} V_n\, dA}_{\substack{\text{FLUSSO QUANTITÀ DI} \\ \text{MOTO SUL CONTORNO}}} = \underbrace{- \int_A \vec{\Phi}_n\, dA}_{\text{SPINTE}}$$

$$\vec{G} + \underset{\substack{\uparrow \\ \text{Forze di inerzia nulle} \\ \text{per moto permanente}}}{\vec{I}} + \vec{M} + \vec{\Pi} = 0$$

$\boxed{\beta}$

$$M = \int_A \cancel{\rho} \vec{V}^2 dA = \beta \cancel{\rho} V_m^2 A$$

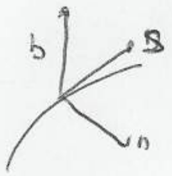
$$\beta = \frac{\int V^2 dA}{V_m^2 A}$$

① FLUIDO
- IDEALE → $\mu = 0$ → $\tau = 0$ → Non perde energie lungo il suo moto
- 1° ORDINE
- T cost

$$\boxed{\rho(\vec{f} - \vec{a}) = \text{grad}(p)}$$

FORMULA DI EULERO

$$\begin{cases} \rho(f_x - a_x) = \frac{\partial p}{\partial x} \\ \rho(f_y - a_y) = \frac{\partial p}{\partial y} \\ \rho(f_z - a_z) = \frac{\partial p}{\partial z} \end{cases}$$



② FLUIDO PESANTE ⇒ $f = -g\,\text{grad}\,z$

componente s

acc tangenziale

acc normale

acc bin

$$\begin{cases} \rho\left(\frac{-g\,\text{grad}\,z}{\partial s} - \frac{a}{\partial s}\right) = \frac{\partial p}{\partial s} \\ \rho\left(\frac{-g\,\text{grad}\,z}{\partial n} - \frac{a}{\partial n}\right) = \frac{\partial p}{\partial n} \\ \rho\left(\frac{-g\,\text{grad}(z)}{\partial b} - \frac{a}{\partial b}\right) = \frac{\partial p}{\partial b} \end{cases} \Rightarrow \begin{cases} \rho\left(-g\frac{\partial z}{\partial s} - \frac{dV}{dt}\right) = \frac{\partial p}{\partial s} \\ \rho\left(-g\frac{\partial z}{\partial n} - \frac{V^2}{r}\right) = \frac{\partial p}{\partial n} \\ \rho\left(-g\frac{\partial z}{\partial b}\right) = \frac{\partial p}{\partial b} \end{cases}$$

$$\Rightarrow \begin{cases} +\gamma\frac{\partial z}{\partial s} + \rho\frac{dV}{dt} - \frac{\partial p}{\partial s} = 0 \\ +\gamma\frac{\partial z}{\partial n} + \rho\frac{V^2}{r} - \frac{\partial p}{\partial n} = 0 \\ +\gamma\frac{\partial z}{\partial b} - \frac{\partial p}{\partial b} = 0 \end{cases}$$

③ FLUIDO INCOMPRIMIBILE

⇓

$\rho = \text{cost}$

$\rho g = \gamma = \text{cost}$

$$\begin{cases} \frac{\gamma}{\gamma}\frac{\partial z}{\partial s} + \frac{\rho}{\gamma}\frac{dV}{dt} - \frac{1}{\gamma}\frac{\partial p}{\partial s} = 0 \\ \frac{\gamma}{\gamma}\frac{\partial z}{\partial n} + \frac{\rho}{\gamma}\frac{V^2}{r} - \frac{1}{\gamma}\frac{\partial p}{\partial s} = 0 \\ \frac{\gamma}{\gamma}\frac{\partial z}{\partial b} + -\frac{1}{\gamma}\frac{\partial p}{\partial b} = 0 \end{cases}$$

$$\to \begin{cases} \frac{\partial}{\partial s}\left(z + \frac{p}{\gamma}\right) = -\frac{1}{g}\frac{dV}{dt} \\ \frac{\partial}{\partial n}\left(z + \frac{p}{\gamma}\right) = -\frac{1}{g}\frac{V^2}{r} \\ \frac{\partial}{\partial b}\left(z + \frac{p}{\gamma}\right) = 0 \end{cases}$$

→ la $\Delta E_p = \Delta V / g$ $\left[\frac{m}{s}\frac{s^2}{m}\right]$

→ la quota piezometrica è inversamente prop alla accelerazione centripeta

↳ diminuisce verso il centro di curvatura

→ V_A A 0 e diviene idrostatica se $r = \infty$

↳ lungo la binormale ho statica!! - distribuzione idrostatica

④ LUNGO LA DI S

$V(t, s(t))$

$$\frac{d\bar{V}}{dt} = \frac{\partial \bar{V}}{\partial t} + \frac{\partial \bar{V}}{\partial s}\frac{ds}{dt} = \frac{\partial \bar{V}}{\partial t} + \frac{\partial \bar{V}}{\partial s}V = \frac{\partial \bar{V}}{\partial t} + \frac{\partial}{\partial s}\left(\frac{V^2}{2}\right)$$

$$\boxed{\frac{\partial(V^2)}{\partial s} = 2V\frac{\partial V}{\partial s}}$$

$$\frac{\partial}{\partial s}\left(z + \frac{p}{\gamma}\right) = -\frac{1}{g}\frac{\partial V}{\partial t} + \frac{\partial}{\partial s}\left(\frac{V^2}{2}\right)$$

$$\frac{\partial}{\partial s}\underbrace{\left(z + \frac{p}{\gamma} + \frac{V^2}{2g}\right)}_{E\text{ per unità di peso}} = -\frac{1}{g}\frac{\partial V}{\partial t}$$

⑤ MOTO PERMANENTE

↳ $\frac{\partial V}{\partial t} = 0$ → $\frac{\partial H}{\partial s} = 0$

ma

BERNULLI

$$\boxed{z + \frac{p}{\gamma} + \frac{V^2}{2g} = \text{cost}}$$

1. PESANTE
2. MOTO PERMANENTE
3. FLUIDO IDEALE
4. INCOMPRIMIBILE
5. correnti lineari o gradualmente variate

$V_m = \frac{Q}{A}$

CORRENTI IN TUBO DI FLUSSO (?)

$$\frac{H \cdot \gamma W}{t} = \frac{E_{me}}{t} \Rightarrow dP = H\gamma dQ$$

$$P = \gamma\int\frac{V^3}{2g}dA = \gamma\alpha\left(\frac{V_m^3}{2g}\right)$$

$$P = \gamma\int_A\left(z + \frac{p}{\gamma} + \frac{V^2}{2g}\right)V\,dA + \gamma\int\frac{V^3}{2g}dA \Rightarrow P = \gamma\left(z + \frac{p}{\gamma}\right)Q + \gamma\alpha\frac{V_m^2}{2g}Q = \text{cost}$$

P cinetica

$$\boxed{z + \frac{p}{\gamma} + \alpha\frac{V_m^2}{2g} = \text{cost}}$$

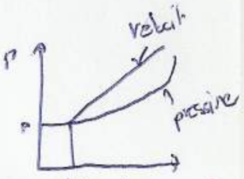
E si conserva se il moto è rettilineo

APPLICAZIONE

① TORNADO

$\rho(\vec{F}-\vec{a}) = \text{grad}\,\vec{p}$

lungo $\vec{n}$: $-\frac{1}{g}\frac{v^2}{r} = \frac{\partial}{\partial n}\left(\frac{p}{\gamma}\right)$ = ABBIAMO r e n opposte → cambio segno → $\frac{\partial}{\partial n}\left(\frac{p}{\gamma}\right) = \frac{1}{g}\frac{v^2}{n}$



$\rightarrow \frac{\partial}{\partial n}(p) = \frac{\gamma}{g}\frac{v^2}{n} = \int_{p_0}^{p} dp = \rho\int_{r_0}^{r}\frac{V^2}{n}$ ← $c_1 r_1$ - lineare → $p(r)-p_0 = \rho\frac{c_1^2 r_1^2}{2}\Big|_{r_0}^{r}$ $\boxed{p(r) = p_0 + \frac{\rho c_1^2}{2}(r^2-r_0)^2}$

velocità lineare ma pressione quadratica

② TUBO

$$\begin{cases} \frac{\partial}{\partial s}\left(z+\frac{p}{\gamma}\right) = -\frac{1}{g}\frac{dV}{dt} \\ \frac{\partial(\frac{p}{\gamma}+z)}{\partial n} - \frac{1}{g}\frac{V^2}{r} \\ \frac{\partial(z+\frac{p}{\gamma})}{\partial b} \quad 0 \end{cases}$$

→ se rettilineo $r = \infty$

$$\begin{cases} \frac{\partial}{\partial s}\left(z+\frac{p}{\gamma}\right) = -\frac{1}{g}\frac{dV}{dt} \\ \frac{\partial(z+\frac{p}{\gamma})}{\partial n} = 0 \\ \frac{\partial}{\partial b}\left(z+\frac{p}{\gamma}\right) = 0 \end{cases}$$

→ la pressione varia con la variazione di velocità

→ PRESSIONE IDROSTATICA

PROCESSO DI EFFLUSSO

velocità torricelliana

molto tent → foro piccolo

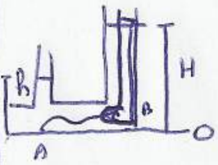
→ SAGOMATA / CORRENTE LINEARE

$$z_A + \cancel{\frac{p_A}{\gamma}} + \cancel{\frac{V_A^2}{2g}} = z_B + \cancel{\frac{p_B}{\gamma}} + \frac{V_B^2}{2g}$$

$V_B^2 = 2g(z_A - z_B) \rightarrow V_B = \sqrt{2g(z_A - z_B)} \rightarrow V_B = \overset{\text{perdite}}{C_v}\sqrt{2g(z_A - z_B)}$

$Q = C_c A V = A C_c C_v \sqrt{2g(z_A - z_B)} = \boxed{\mu A\sqrt{2g(z_A - z_B)} = Q}$

TUBO DI PITAUX



$\cancel{z_A} + \frac{p_A}{\gamma} + \frac{V_A^2}{2g} = z_B + \frac{p_B}{\gamma} + \cancel{\frac{V_B^2}{2g}}$ → P.TO DI RISTAGNO

$h + \frac{V_A^2}{2g} = H$

$V_A^2 = \sqrt{(H-h)2g} = \sqrt{2g\Delta}$

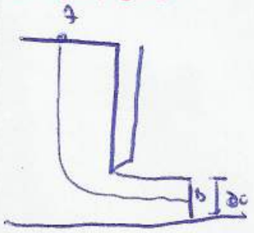
p

v≠

⇓

ρ≠ ⇒ dato e

SARACINESCA



$z_A = \underbrace{z_B + \frac{p_A}{\gamma}}_{a_c} + \frac{V_A^2}{2g}$ ⇒ $V_B\; C_v\sqrt{2g(z_2 - aC_c)}$

coef. di contrazione

$A C_c = a_c b \rightarrow a b C_c = a_c b \rightarrow a_c = a C_c$

$\rho(\vec{F}-\vec{a}) = \nabla p$ $\quad \frac{dV}{dt} = \cancel{\frac{\partial V}{\partial t}} + \frac{\partial}{\partial s}\frac{V^2}{2}$

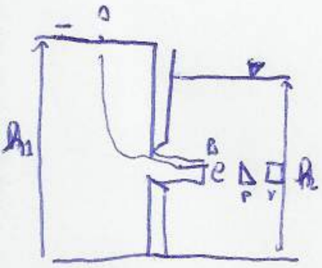
lungo s → $\rho\left(g\frac{\partial z}{\partial s} - \frac{dV}{dt}\right) = \frac{\partial p}{\partial s}$

$\gamma\frac{\partial z}{\partial s} - \rho\frac{dV}{dt} = \frac{\partial p}{\partial s}$

$\frac{\partial}{\partial s}\left(gz + \frac{V^2}{2}\right) + \frac{1}{\rho}\frac{\partial p}{\partial s} = 0$

$$\boxed{gz + \frac{V^2}{2} + \int\frac{dp}{\rho} = cost}$$

EFFLUSSO INTERNO



$z_A + \frac{p_A}{\gamma} + \cancel{\frac{V_A^2}{2g}} = \underbrace{z_B + \frac{p_B}{\gamma}} + \frac{V_B^2}{2g}$

$h_1 = h_2 + \frac{V_B^2}{2g} \rightarrow V_B = C_v\sqrt{2g(h_1 - h_2)}$

FLUIDO COMPRIMIBILE

→ ARIA sotto i 300 $\frac{km}{h}$ è incomprimibile

$pW^n = cost$ legge dei gas $\rightarrow p\left(\frac{M}{\rho}\right)^n = cost \rightarrow \frac{p}{\rho^n} = cost \rightarrow \frac{p}{cost} = \rho^n \rightarrow \int\frac{dp}{\rho} = cost^{\frac{1}{n}}\int\frac{dp}{p^{\frac{1}{n}}}$

- $n = 1$ $\quad c\int\frac{dp}{p} = \boxed{c\ln p}$
- $n \neq 1$ $\quad c^{\frac{1}{n}}\int p^{-\frac{1}{n}}dp = c^{\frac{1}{n}}\frac{p^{1-\frac{1}{n}}}{1-\frac{1}{n}} = \frac{n}{n-1}\frac{1}{\rho}p = \boxed{\frac{n}{n-1}\frac{p}{\rho}}$ $\quad \frac{1}{\rho}$

$\rho = cost$

$gz + \frac{V_1^2}{2} + \frac{p_1}{\rho} = cost$

FLUIDI COMPRIMIBILI

$$gz + \frac{V^2}{2} + \int \frac{dP}{\rho} = cost \quad \begin{cases} n=1 & = c \ln p \\ n \neq 1 & = \frac{n}{n-1} \frac{P}{\rho} \end{cases}$$

$\boxed{cs}$ $n \neq 1$

$\boxed{\rho = cost}$ $\boxed{\forall n}$ → benulli $= \overset{z_1 = z_2}{\cancel{z}} + \frac{P_1}{\gamma} + \frac{V_1^2}{2g} = \cancel{z} + \frac{P_2}{\gamma} + \overset{\text{PUNTO DI RISTAGNO}}{\cancel{\frac{V^2}{2g}}}$ $\quad P_2 - P_1 = \gamma \frac{V_1^2}{2g} = \boxed{P_2 - P_1 = \frac{V_1^2}{2} \rho}$

C = celerità = velocità di propagazione di una perturbazione nel fluido $= \sqrt{\frac{\varepsilon}{\rho}}$ = in un fluido ideal $c = \sqrt{\frac{n P_2}{\rho}}$ - $C_1^2 = \frac{nP_1}{\rho}$ $\boxed{\frac{P_1}{\rho} = \frac{C_1^2}{n}}$ $\boxed{\Pi_\sigma = \frac{V}{C}}$

$$P_2 - P_1 = \rho \frac{V_1^2}{2} \rightarrow \frac{P_2 - P_1}{P_1} = \frac{\overset{n/C_1^2}{\rho} V_1^2}{2 P_1} \rightarrow \boxed{\frac{P_2 - P_1}{P_1} = \frac{n}{2} \Pi_\sigma^2}$$ $\rho = cost$ $\forall n$

$\boxed{n=1 \quad \rho \neq cost}$

$$\cancel{z} + \frac{V_1^2}{2} + c \ln P_1 = \cancel{z} + \cancel{\frac{V_2^2}{2}} + c \ln P_2 \rightarrow \ln \frac{P_2}{P_1} = \frac{1}{2C} V_1^2$$

$\boxed{C = \frac{P}{\rho}}$ $\boxed{\Pi_\sigma = \frac{c}{V}}$ $\quad C_1 = \sqrt{\frac{P}{\rho}}$ $\quad C_1^2 = C^2$

$$\Rightarrow \ln \frac{P_2}{P_1} = \frac{1}{2} \overbrace{\frac{V_1^2}{C_1^2}}^{\Pi_2^2} \rightarrow \boxed{\frac{P_2 - P_1}{P_1} = e^{\frac{\Pi_\sigma^2}{2}} - 1}$$ $\rho \neq cost$ $n = 1$

$n \neq 1 \quad \rho \neq cost$

$$\frac{V_1^2}{2} + \frac{n}{n-1} \frac{P_1}{\rho_1} = \frac{n}{n-1} \frac{P_2}{\rho_2} \rightarrow \frac{P_2}{P_1} = \frac{\rho_2}{\rho_1} + \frac{n-1}{n} \frac{V_1^2}{2} \frac{\rho_2}{P_1}$$

① $\boxed{C_1^2 = \frac{P}{\rho} n}$
② $\boxed{\frac{\rho_2^n}{\rho_1^2} = \frac{P_2}{P_1}}$

$$\boxed{\frac{P_2 - P_1}{P_1} = \left(1 + \frac{n-1}{2} \Pi_2^2\right)^{\frac{n}{n-1}} - 1}$$ $\rho \neq cost$ $n = 1$

POTENZA CINETICA IN UN TUBO DI FLUSSO

$H \leftarrow E$ per unità di peso

$H \gamma W = E$

$H \frac{\gamma W}{T} =$ Potenza

$H \gamma Q = P \rightarrow H \gamma \, dQ = dP \rightarrow P = \gamma \int_A z + \frac{P_2}{\gamma} + \frac{V^2}{2g} \, dQ \rightarrow \gamma \int_A \left(z + \frac{P}{\gamma} + \frac{V^2}{2g}\right) V \, dA$

→ POTENZA CINETICA $= \gamma \int_A \frac{V^3}{2g} dA = \alpha \int \frac{V_m^3}{2g}$ $\qquad \alpha = \frac{\int V^3 dA}{V_m^3 A}$ ← Varia con il profilo di V

$$P = \gamma \int_A \left(z + \frac{P}{\gamma}\right) V \, dA + \gamma \alpha \frac{V_m^3 A}{2g}$$

UN FLUIDO IDEALE $\rho = cost$ $P = cost$

$$\cancel{\gamma} Q \boxed{\left(\underbrace{z + \frac{P}{\gamma}}_{cost} + \frac{\alpha V_m}{2g}\right) = cost}$$ E si conserva

- incomprimibile
- moto rettilineo → $\left(z + \frac{P}{\gamma}\right) = cost$

⇒ SI ESTENDE IL TH DI BERNULLI PER UN TUBO DI FLUSSO (NON TRAIETTORIA)

EQUAZIONE GLOBALE PER L'EQ DINAMICO DI UN FLUIDO

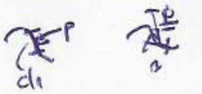
$$\int_W \rho \vec{F}\, dW + \int_A \Phi_n\, dA + \int_A \rho \vec{V} V_n\, dA + \int_W \frac{\partial}{\partial t}(\rho \vec{V})\, dW = 0$$

NON VARIA PER FLUIDI IDEALI

IDEA — REA

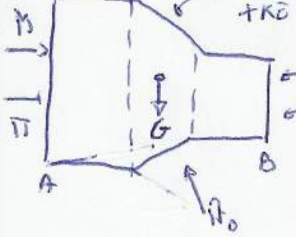
IL FLUSSO DI Q DI MOTO NON VARIA

NON VARIA



ESERCIZI

NON SI NOTA VICINO +KÈ le correnti non sono linea



SPINTE DINAMICHE

$$\overline{M}_A + \overrightarrow{M}_B + \overrightarrow{G} + \overrightarrow{\Pi}_A + \overrightarrow{\Pi}_B + \overrightarrow{\Pi}_0 = 0$$

$\Pi_1 = \gamma h_A A_A$ $\quad M_1 = \rho \bar{V} \overset{Q}{V_n A} = \rho Q V_n = \rho Q V_1$

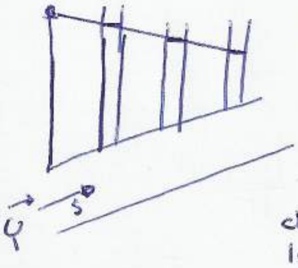
$\Pi_2 = \gamma h_B A_B$ $\quad M_2 = \rho Q V_2$

BILANCIO DI ENERGIA → $z_1 + \frac{p_1}{\gamma} + \alpha \frac{V_{m1}}{2g} = z_2 + \frac{p_2}{\gamma} + \alpha \frac{V_{m2}}{2g}$

" DI MASSA → $Q_1 = Q_2$ $\quad V_1 A_1 = V_2 A_2$

EQUILIBRIO → $\overrightarrow{M}_1 + \overrightarrow{M}_2 + \overrightarrow{\Pi}_1 + \overrightarrow{\Pi}_2 + \overrightarrow{G} = 0$

FLUDI REALI



$H \neq$ cost → viscosità → τ → calore

Ho una perdita di E per unità di peso e lunghezza = J

variazione di E di peso lungo S → $\frac{dH}{ds} = -J \to \int_1^2 dH = \int_1^2 -J\, ds \to H_2 - H_1 = -JL$ $\quad H_2 = H_1 - \overbrace{JL}^{\text{CADUTE PIEZOMETRICHE}}$

REALE → $H_2 = H_1 - JL - \sum \Delta H$ ← RESTRINGIMENTI ETC...

TEOREMA Π

DATA UNA FUNZIONE con n+1 VARIABILI ∃ una base di K elementi.
Adimensionalizzando posso ridurre a n+1-K incognite

$$G = f\Big(\overbrace{l, \underset{\substack{\text{coef}\\\text{ADIM}\\\text{FORMA}}}{\theta}, \underset{\text{RUGOSITÀ}}{n}}^{\text{Geom}}, \overbrace{\rho, \mu, \underset{\text{comprimibilità}}{\varepsilon}, \underset{\substack{\text{TENSIONE}\\\text{SUPE}}}{S}}^{\text{FLUIDO}}, \overbrace{v, a}^{\text{cinematic}}, \overset{\text{tempo}}{t}, \overset{\text{spazio}}{\vec{x}}, g, \overset{\text{Forzante}}{\vec{f}}\Big)$$

SCELGO LA BASE
- L → lunghezza
- M → ρ → centrale nei moti turbolenti
- T → V → presente nei moti stazionari

$$\Pi_G = f\left(1, \theta, \frac{v}{l}, 1, Re, Ma, We, 1, \frac{a l}{v^2}, St, \frac{\bar{x}}{l}, Fr, \overline{\Pi}_F\right)$$

NUMERI ADIMENSIONALI

Reinolds $[\nu]$ $\frac{K_g}{s m}$

[EFFETTI VISCOSI]

$\mu = [M][L^{-1}][T^{-1}]$ $K_g \frac{\cancel{m}}{s^2} \frac{s}{m^4}$

$\mu = \rho^\alpha l^\beta v^\gamma = M L^{-1} T^{-1} = [M L^{-3}]^\alpha [L]^\beta [\cancel{M} L T^{-1}]^\gamma$

$\frac{\text{FORZE INERZIALI}}{\text{FORZE VISCOSE}}$

$$\begin{cases} \alpha = 1 \\ -3\alpha + \beta + \gamma = -1 \\ -\gamma = -1 \end{cases} \rightarrow \begin{cases} \alpha = 1 \\ \beta = 1 \\ \gamma = 1 \end{cases} \qquad \dot{\mu} = \frac{\mu}{\rho v l} \Rightarrow \boxed{R_e = \frac{\rho v l}{\mu}}$$

Cauchy $[\varepsilon]$ $\frac{K_g}{m s^2}$

$dp = \varepsilon \frac{dV}{V} \rightarrow [N][L^{-2}] \quad [M][L][T^{-2}][L^{-2}]$

[FLUIDI COMPRIMIBILI]

$[\varepsilon] = [M]^\alpha [L^{-1}]^\beta [T^{-2}]^\gamma = \rho^\alpha l^\beta \cdot v^\gamma = [M L^{-3}]^\alpha [L]^\beta [L T^{-1}]^\gamma$

$$\begin{cases} \alpha = 1 \\ -3\alpha + \beta + \gamma = -1 \\ -\gamma = -2 \end{cases} \rightarrow \begin{cases} \alpha = 1 \\ \beta = 0 \\ \gamma = 2 \end{cases} \qquad \dot{\varepsilon} = \frac{\varepsilon}{\rho v^2} \qquad \boxed{C_a = \frac{\rho v^2}{\varepsilon}}$$

$$C_a = \frac{\rho V^2}{\varepsilon} = \frac{V^2}{\frac{\varepsilon}{\rho}} = \boxed{\frac{V^2}{c^2} = M_a^2}$$

WEBER $[s] = M T^{-2}$

$\frac{N}{m}$

[SE HO + FLUIDI, CURVATURA, Tensione superficiale]

$$\begin{cases} \alpha = 1 \\ -3\alpha + \beta + \gamma = 0 \\ -\gamma = -2 \end{cases} \rightarrow \begin{cases} \alpha = 1 \\ \beta = 1 \\ \gamma = 2 \end{cases} \qquad We = \frac{s}{\rho l v^2} = \frac{\rho V^2}{\frac{s}{l}}$$

ρV^2 ← SFORZO TURBOLENTO

$\frac{s}{l}$ ← sforzo di tensione sup

$[a] = [L][T]^{-2}$

[MOTI NON STAZIONARI]

$$\begin{cases} \alpha = 0 \\ \beta = -1 \\ \gamma = 2 \end{cases} \qquad \dot{a} = \frac{a l}{v^2}$$

STROUHAL $[T]$

[MOTI NON STAZIONARI]

$[t] = T$

$$\begin{cases} \alpha = 0 \\ \beta = 1 \\ \gamma = 1 \end{cases} \qquad \dot{S}t = \frac{t}{\frac{l}{v}} \Rightarrow St = \frac{\frac{l}{v}}{t}$$

$\frac{l}{v}$ ← tempo medio di passaggio

t ← tempo che studio

FRUD $[g]$

$$\dot{g} = \frac{g l}{v^2} \qquad Fr^2 = \frac{v^2}{g l} \frac{\rho}{\rho}$$

SFORZI DI NATURA TURBOLENTA

SFORZI PER FORZA PESO

[SOLO SE HO + FLUIDI CHE LA GRAVITÀ DA INTERAZIONI DIFFERENTI]

SIMILITUDINI

- Geometrica $\frac{L'}{L} = \text{cost} = \lambda_L$
- Cinematica $\frac{v'}{v} = \text{cost} = \lambda_v$ $\frac{a'}{a} = \text{cost} = \lambda_a$ $\frac{T'}{T} = \frac{v}{L}\frac{L'}{v'} = \frac{\lambda_L}{\lambda_v}$
- DINAMICA $\frac{F'}{F} = \text{cost} =$!

CONDIZIONE SUFF. ① $\frac{F'}{F} = \lambda_F \Rightarrow \frac{m'a}{ma} = \frac{\rho' w'}{\rho w}\lambda_a = \lambda_\rho \lambda_\ell^3 \lambda_a = \lambda_\rho \lambda_\ell^2 \lambda_v^2$

② TUTTI I PARAMETRI DI CONTROLLO SONO SIMILI

SIMILITUDINI PARZIALI PER CUI COINCIDONO LE VARIABILI BASE DEL SISTEMA

AUTOSIMILE

SLIDE

MODELLI FLUIDODINAMICI

$$G = f\left(Re, Ma, We, Fr, \theta, \frac{\overset{\varepsilon}{r}}{\ell}, \frac{\overset{a}{a}\ell}{v^2}, \quad , \frac{x}{L}, \frac{y}{L}, \frac{z}{L}, St\right)$$

① SCALA GEOM $L'/L = \lambda_L = \lambda$

② ACC GRAVITÀ $g'/g = 1 = \lambda_g$ → sono sulla terra

③ FLUIDO $\rho'/\rho = \lambda_\rho = 1$

$\mu'/\mu = \lambda_\mu = 1$ → Non cambio il fluido

$\varepsilon'/\varepsilon = \lambda_\varepsilon = 1$

$s'/s = \lambda_s = 1$

6 VINCOLI per 3 GDL

Modello in similitudine di Reynold

$Re' = Re \Rightarrow \lambda_{Re} = 1$ $\frac{\rho v \ell}{\mu}\frac{\mu'}{\rho' v' \ell'} = 1$ $\frac{v'}{v} = \frac{\ell}{\ell'} = \boxed{\lambda_v = \frac{1}{\lambda_L}}$

$Ma' = Ma$ $\frac{v'}{c'} = \frac{v}{c} \Rightarrow \frac{v'}{\sqrt{\frac{\varepsilon'}{\rho'}}} = \frac{v}{\sqrt{\frac{\varepsilon}{\rho}}} \Rightarrow \boxed{\lambda_v = 1}$

Se $\lambda_v = 1 \Rightarrow \lambda_L = 1$

ma $\lambda_L = \lambda \neq 1$

⇓

SIMILITUDINE INCOMPATIBILE

SIMILITUDINE DI REYNOLDS

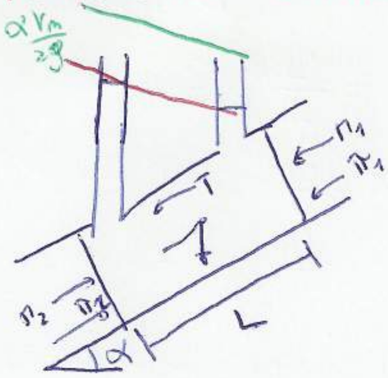
$\lambda_{Re} = 1 \Rightarrow \lambda_v = \frac{1}{\lambda_L}$ se pongo $\lambda_L = \frac{1}{10}$ $\lambda_v = \frac{1}{\cancel{10}} 10$

$\frac{\lambda_L}{\lambda_v} = \lambda_T = \frac{1}{100}$ $\lambda_a = \frac{\lambda_v^2}{\lambda_L} = \frac{1}{\lambda_L}^3 = \frac{1}{1000}$

$\lambda_F = \frac{m'a'}{ma} = \frac{\rho' L'^3 a'}{\rho L^3 a} = 1 \; \lambda_L^3 \lambda_a = 1 \Rightarrow$ SIMILITUDINI DI REYNOLDS STESSE FORZE

$\lambda_{Ma} = \frac{v'}{\varepsilon'}\frac{\varepsilon}{v} = \frac{1}{\lambda_L}$ → il mio modello a un Ma 10 volte sup → NON BENE

PERDITE DI CARICO IN PROBLEMI MONO-DIMENSIONALI



$$\frac{\partial H}{\partial s} = -J$$

$$\tau = f(D, \cancel{\ell}, r, \rho, \mu, \cancel{E}, \cancel{\beta}, v, \cancel{a}, \cancel{x}, \cancel{\gamma}, \cancel{\delta}, \cancel{F})$$

per unità di lunghezza

INC.

NO T SUP

NO ACC

1 FLUIDO

NO FUTA

NON DIPENDE dalla posizione

STAZIONARIO

BASE
- LAM (ρ, V, D) ②
- TURB $(\cancel{\mu}, V, D)$ ①

① $\frac{\tau}{\rho V^2} = f_1\left(1, \frac{r}{D}, 1, Re, 1\right)$

② $\frac{\tau}{\mu \frac{V}{D}} = f_2\left(1, \frac{r}{D}, Re, 1, 1\right)$

LUNGO S

$$\vec{\Pi}_1 + \vec{\Pi}_2 + \vec{M}_1 + \vec{M}_2 + \vec{T} + \vec{G}_{//} = 0 \quad \rightarrow \quad P_1 A - P_2 A + \gamma w \sin\alpha - |\vec{T}| = 0$$

$$P_1 A - P_2 A - \gamma A (z_2 - z_1) - |\vec{T}| = 0$$

$$\vec{T} = A\gamma \underbrace{\left[\left(\frac{P_1}{\gamma} + z_1\right) - \left(\frac{P_2}{\gamma} + z_2\right)\right]}$$

$$\vec{T} = A\gamma \quad LJ$$

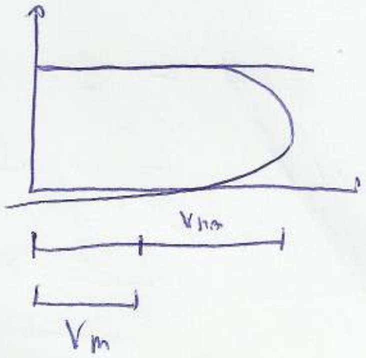
PER TUTTI I MOTI

$$J = \frac{\vec{T}}{AL\gamma} = \frac{\tau \pi D \cancel{L}}{\gamma A \cancel{L}} = \frac{\tau \cancel{\pi} \cancel{D}}{\gamma \cancel{\pi} \frac{D^{\cancel{2}}}{4}} = \boxed{\frac{4\tau}{\gamma D}}$$

$$J = \begin{cases} \frac{4\tau}{\gamma D} \overset{LAM}{=} \frac{4}{\gamma D} \rho V^2 f\left(\frac{r}{D}, Re\right) = \overbrace{8 f_1\left(Re, \frac{r}{D}\right)}^{\lambda \text{ TURBOLENTO}} \frac{V^2}{2gD} \\ \frac{4\tau}{\gamma D} \overset{TURB}{=} \frac{4}{\gamma D} \frac{\mu V}{D} f\left(\frac{r}{D}, Re\right) = \overbrace{\frac{8}{Re} f_2\left(Re, \frac{r}{D}\right)}^{\lambda \text{ LAMINARE}} \frac{V^2}{2gD} \end{cases}$$

α e β nel moto laminare

$$\alpha = \frac{\int_A V^3 dA}{V_m^3 A} = \frac{\frac{2\cancel{8} \cancel{V^3} A}{\cancel{4}}}{\cancel{V^3} A} = 2$$

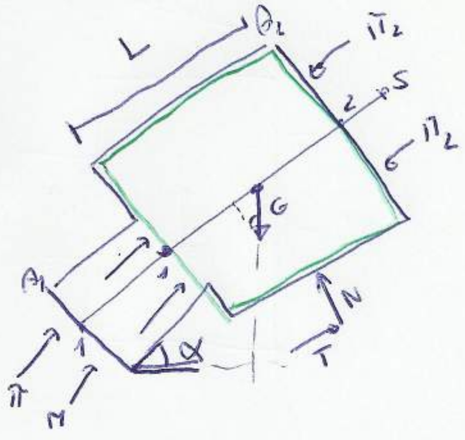


$$\beta = \frac{\int V^2 dA}{V_m^2 A}$$

$$\beta = \frac{4 \frac{\cancel{V_m^2}}{3} \cancel{A}}{\cancel{V_m^2 A}} = \frac{4}{3}$$

PERDITE LOCALI

BRUSCO ALLARGAMENTO



$$\vec{M}_1+\vec{\Pi}_1+\vec{\Pi}_2+\vec{M}_2+\vec{G}+\vec{T}+\vec{N}=0$$

$$M_1+\Pi_1-M_2-\Pi_2-G\sin\alpha+T=0$$

$$\rho QV_1+p_1A_1+\rho QV_2-p_2A_2-\cancel{\gamma}A_2\overbrace{L\sin\alpha}^{h=l_1}+\cancel{T}=0$$ → TRASCURABILE

$$\rho Q(V_1-V_2)+p_1A_2-p_2A_2-\cancel{\gamma}A_2(z_2-z_1)=0$$

$$\rho Q(V_1-V_2)+A_2\left[p_1-p_2-\cancel{\gamma}(z_2-z_1)\right]=0$$

$$\rho Q(V_1-V_2)=\gamma A_2\left[-\frac{p_1}{\gamma}-z_1+\frac{p_2}{\gamma}+z_2\right]$$

$$\rho Q(V_1-V_2)=\gamma A_2\left[\left(\frac{p_2}{\gamma}+z_2\right)-\left(\frac{p_1}{\gamma}+z_1\right)\right]$$ → LA LINEA PIEZOMETRICA NON STA [illegible]

$$\Delta H= z_1+\frac{p_1}{\gamma}+\frac{V_1}{2g}=z_2+\frac{p_2}{\gamma}+\frac{V_2}{2g}$$

$$\Delta H=\underbrace{\left(z_1+\frac{p_1}{\gamma}\right)-\left(z_2+\frac{p_2}{\gamma}\right)}+\frac{1}{2g}\left(V_1^2-V_2^2\right)$$

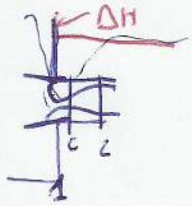
$$\Delta H=\frac{\cancel{\rho}\,\overset{V_2\cancel{A_2}}{\cancel{Q}}(V_1-V_2)}{g\cancel{\gamma}\cancel{A_2}}+\frac{1}{2g}\left(V_1^2-V_2^2\right)\longrightarrow\frac{V_2(V_1-V_2)}{g}+\frac{1}{2g}\left(V_1^2-V_2^2\right)=\Delta H$$

$$\Delta H=\frac{1}{2g}\left(-2V_2V_1+2V_2^2+V_1^2-V_2^2\right)=\boxed{\frac{1}{2g}\overset{\text{coef.}}{\alpha}(V_1-V_2)^2}$$

CASO PARTICOLARE: ingresso serbatoio

$$\frac{1}{2g}\alpha(V_1-0)^2\Rightarrow$$ perdo tutto il termine cinetico

IMBOCCO



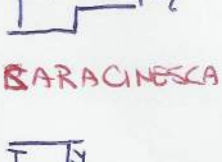
$$\Delta H_{c1}=\frac{V^2}{2g}\simeq 0{,}1\frac{V^2}{2g}$$

$$Q_I=Q_{II}\rightarrow V_cA_c=VA\rightarrow V_c\cancel{A_c}=V\frac{\cancel{A_c}}{c_c}\quad\boxed{V_c=\frac{V}{C_c}}$$

$$\Delta H_{c2}=\frac{1}{2g}(V_c-V_2)^2\longrightarrow\frac{V^2}{2g}\left(\frac{V_c}{V}-1\right)^2\rightarrow\Delta H_{c2}=\frac{V^2}{2g}\left(\frac{1}{c_c}-1\right)^2\simeq 0{,}4\frac{V^2}{2g}$$

$$\Delta H=\Delta H_{c1}+\Delta H_{c2}=\boxed{\frac{1}{2}\frac{V^2}{2g}}$$

BRUSCO RESTRINGIMENTO

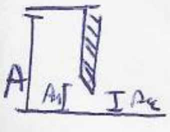


$$\Delta H=\frac{nV^2}{2g}$$ / $n=0{,}5$ $D_1>2D_2$

DIV

$$\Delta H=n(\beta)\frac{1}{2g}\left(V_1-V_2\right)^2$$

BARACINESCA



$$\Delta H=\frac{1}{2g}\left(V_c-V\right)^2$$

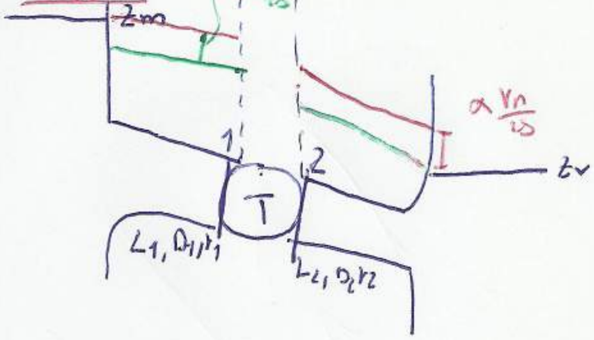
$$Q_e=Q_u$$

$$V_1A=V_cA_1c_c$$

$$V_c=\frac{VA}{c_cA_1}\quad\frac{V}{c_cm}$$

MACCHINE IDRAULICHE

TURBINA



LA TURBINA È "STUDIABILE" COME UNA PERDITA

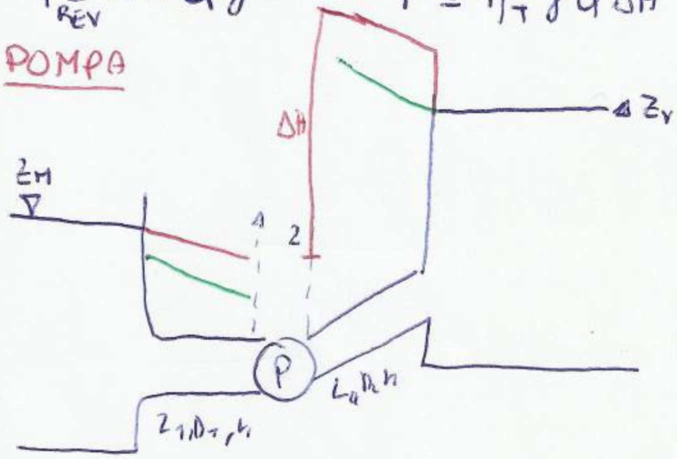
E viene ceduta alla turbina dal fluido

$$L_T = \Delta H = H_1 - H_2 \Rightarrow z_m - J_1 L_1 - \left(z_v + J_2 L_2 \right) + \alpha \frac{V_2^2}{2g})$$

$$L_T = z_m - J_1 L_1 - z_v - J_2 L_2 - \alpha \frac{V_2^2}{2g} \Rightarrow \overbrace{(z_m - z_v)}^{\text{SALTO GEODETICO}} - \overbrace{\left(J_1 L_1 + J_2 L_2 + \alpha \frac{V_2^2}{2g}\right)}^{\text{perdite}}$$

$$P_{REV} = \Delta H Q \gamma \qquad P = \eta_T \gamma Q \Delta H$$

POMPA



$$\Delta H = H_2 - H_1$$

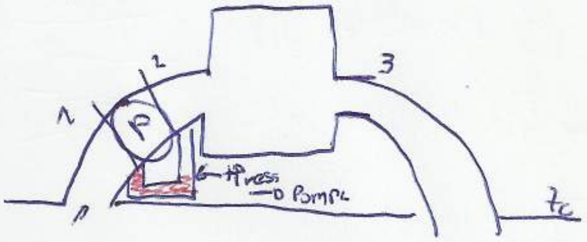
$$\Delta H = -\left(z_M - J_1 L_1\right) + \left(z_v + J_2 L_2 + \alpha \frac{V_2^2}{2g}\right)$$

$$\Delta H = \underbrace{(z_v - z_M)}_{} + \left(J_1 L_1 + J_2 L_2 + \alpha \frac{V_2^2}{2g}\right)$$

FORNISCO E per il salto geodetico + tutte le perdite

$$P_{rev} = \gamma Q \Delta H \qquad P = \frac{\gamma Q \Delta H}{\eta_P}$$

CIRCUITO CHIUSO



$$H_1 = z_c - J_1 L_1 - \overset{\text{perdite im}}{1{,}16 \frac{V^2}{2g}}$$

$$H_3 = z_c + J L_3 + \alpha \frac{V^2}{2g} + \overbrace{\frac{1}{2} \frac{V^2}{2g}}^{\text{SPIGOLO VIVO}}$$

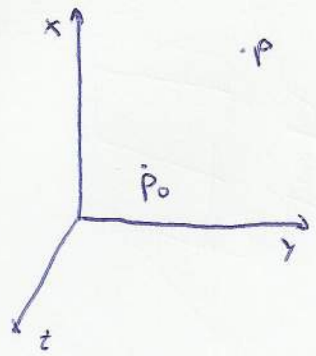
$$H_2 = H_3 + J_2 L_2 + \alpha \frac{V_2}{2g}$$

$$\Delta H = H_2 - H_1 = \cancel{z_c} + J_3 L_3 + \alpha \frac{V^2}{2g} + \frac{V^2}{4g} + J_2 L_2 + \alpha \frac{V_2^2}{2g} - \cancel{z_c} + J_1 L_1 + 1{,}16 \frac{V^2}{2g}$$

$$J(L_1 + L_2 + L_3) + \frac{V^2}{2g}\left(\alpha + 0{,}5 + \alpha + 1{,}16\right)$$

↑ DEVO FORNIRE E SOLO PER VINCERE LE PERDITE

DEFORMAZIONI

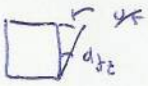


$$\bar{V}(P_0)=\bar{U}_0+\bar{V}_0+\bar{Z}_0$$

$$\bar{V}(P)=\bar{U}_0+\frac{\partial \bar{V}_0}{\partial x}dx+\frac{\partial \bar{V}_0}{\partial y}dy+\frac{\partial \bar{V}}{\partial z}dz$$

$$\begin{cases} U=U_0+\frac{\partial U}{\partial x}dx+\frac{\partial U}{\partial y}dy+\frac{\partial U}{\partial z}dz \\ V=V_0+\frac{\partial V}{\partial x}dx+\frac{\partial V}{\partial y}dy+\frac{\partial V}{\partial z}dz \\ W=W_0+\frac{\partial W}{\partial x}dx+\frac{\partial W}{\partial y}dy+\frac{\partial W}{\partial z}dz \end{cases}$$

NEWTON



$\cancel{y}\ \frac{\partial u}{\partial y}dy\,dt$

$$\frac{\partial u}{\partial y}\cancel{dy}\,dt=d\gamma_t\,\cancel{dy}$$

$$\frac{\partial u}{\partial y}=\frac{\partial \gamma_t}{dt}$$

$$\tau=-\mu\frac{\partial u}{\partial y}=-\mu\frac{\partial \gamma_t}{\partial T}$$

$$\bar{V}=\begin{bmatrix}U\\V\\W\end{bmatrix}\quad \bar{V}_0=\begin{bmatrix}U_0\\V_0\\W_0\end{bmatrix}\quad \bar{dx}=\begin{bmatrix}dx\\dy\\dz\end{bmatrix}\quad \underline{\underline{A}}=\begin{bmatrix}\frac{\partial U}{\partial x}&\frac{\partial U}{\partial y}&\frac{\partial U}{\partial z}\\ \frac{\partial V}{\partial x}&\frac{\partial V}{\partial y}&\frac{\partial V}{\partial z}\\ \frac{\partial W}{\partial x}&\frac{\partial W}{\partial y}&\frac{\partial W}{\partial z}\end{bmatrix}\Rightarrow \boxed{\bar{V}=\bar{V}_0+\underline{\underline{A}}\,\bar{dx}}$$

$$\bar{\bar{A}}=\frac{\bar{\bar{A}}+\bar{\bar{A}}^T}{2}+\frac{\bar{\bar{A}}-\bar{\bar{A}}^T}{2}=\bar{\bar{D}}+\bar{\bar{\Omega}}$$

$$\bar{\bar{A}}_T\begin{bmatrix}\frac{\partial U}{\partial x}&\frac{\partial V}{\partial x}&\frac{\partial W}{\partial x}\\ \frac{\partial U}{\partial y}&\frac{\partial V}{\partial y}&\frac{\partial W}{\partial y}\\ \frac{\partial U}{\partial z}&\frac{\partial V}{\partial z}&\frac{\partial W}{\partial z}\end{bmatrix}$$

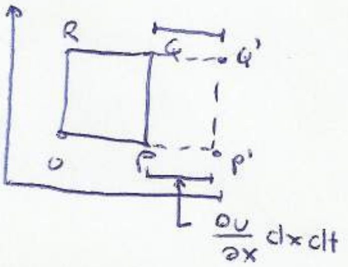
MATRICE DEFORM

$$\bar{\bar{D}}=\begin{bmatrix}\frac{\partial U}{\partial x}&\frac{1}{2}\left(\frac{\partial U}{\partial y}+\frac{\partial V}{\partial x}\right)&\frac{1}{2}\left(\frac{\partial U}{\partial z}+\frac{\partial W}{\partial x}\right)\\ \cdot&\frac{\partial V}{\partial y}&\cdot\\ \cdot&\cdot&\frac{\partial W}{\partial z}\end{bmatrix}\qquad \bar{\bar{\Omega}}=\begin{bmatrix}0&\frac{1}{2}\left(\frac{\partial U}{\partial y}-\frac{\partial V}{\partial x}\right)&''\\ \cdot&0&\cdot\\ \cdot&\cdot&0\end{bmatrix}$$

MATRICE ROTAZIONI

MATRICE DI DEF $[\underline{\underline{D}}]$

DIAGONALE



① $V_0 \Rightarrow U_0=0 \quad V_0=0$

$V_P \Rightarrow U_P=\frac{\partial U}{\partial x}dx \quad V_P=\frac{\partial V}{\partial x}dx$

$V_R \rightarrow U_R=\frac{\partial U}{\partial y}dy \quad V_R=\frac{\partial V}{\partial y}dy$

$V_Q \Rightarrow U_Q=\frac{\partial U}{\partial x}dx+\frac{\partial U}{\partial y}dy \quad V_Q=\frac{\partial V}{\partial x}dx+\frac{\partial V}{\partial y}dy$

$$d\varepsilon_x=\frac{OP'}{OP^\circ}=\frac{\frac{\partial U}{\partial x}\cancel{dx}\,dt}{\cancel{dx}}=\frac{\partial U}{\partial x}dt\rightarrow\boxed{\frac{d\varepsilon_x}{dt}=\frac{\partial U}{\partial x}}$$

IL PRIMO termine mi definisce la velocità di deformazione lungo x

→ ANALOGO ALTRE DIR

$$dW=W_F-W_I=\left(dx+\frac{\partial u}{\partial x}dx\,dt\right)\cdot\left(dy+\frac{\partial v}{\partial y}dy\,dt\right)\cdot\left(dz+\frac{\partial w}{\partial z}dz\,dt\right)-\overbrace{dx\,dy\,dz}^{W_I}$$

$$\frac{dW}{W}=\cancel{1}+\frac{\partial u}{\partial x}dt+\frac{\partial v}{\partial y}dt+\frac{\partial w}{\partial z}dt\rightarrow\boxed{\frac{1}{W}\frac{dW}{dt}=div(\bar{v})}$$

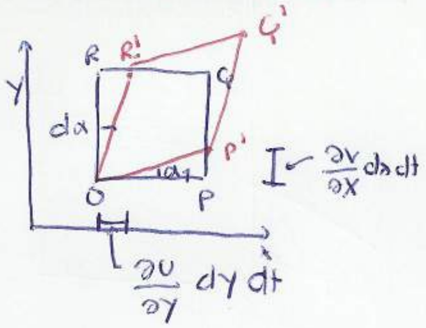
IL PRIMO Termine è la velocità di def di volume unitario

se $\rho=0$ $div(\bar{v})=0$

Nell' eq di continuità

$$\frac{d\rho}{dt}+\rho\,div(\bar{v})=\boxed{\frac{d\rho}{dt}+\frac{\rho}{W}\frac{dW}{dt}=0}$$

TERMINI EXTRADIAGONALI



$$\frac{1}{2}\left(\frac{\partial u}{\partial y} + \frac{\partial u}{\partial x}\right)$$

$$\frac{\partial u}{\partial y}\cancel{dy}\, dt = \cancel{dy}\; tg\,(d\alpha) \qquad \frac{\partial v}{\partial x}\cancel{dx}\, dt = \cancel{dx}\; tg\,(d\alpha_1)$$

$$\boxed{\frac{\partial u}{\partial y} = \frac{d\alpha_2}{dt}} \qquad \boxed{\frac{\partial v}{\partial x} = \frac{d\alpha_1}{dt}}$$

→ VOLUME IDENTICO VARIAZIONE DI FORMA non DI VOLUME

$d\alpha_1 + d\alpha_2 = d\gamma_z$
↳ giace sul piano zeta

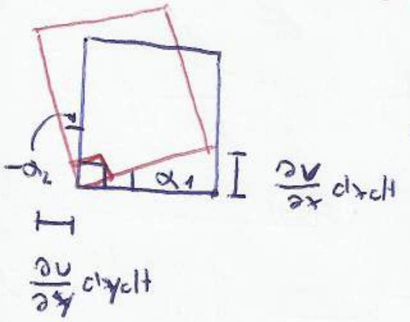
$$d\gamma_z = \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) dt \rightarrow \frac{d\gamma_z}{dt} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}$$

la componente di D mi indica metà delle velocità di deformazione

$$\bar{\bar{D}} = \begin{bmatrix} \frac{d\varepsilon_x}{dt} & \frac{1}{2}\frac{d\gamma_z}{dt} & \frac{1}{2}\frac{d\gamma_x}{dt} \\ '' & \frac{d\varepsilon_y}{dt} & \frac{1}{2}\frac{d\gamma_y}{dt} \\ '' & '' & \frac{d\varepsilon_z}{dt} \end{bmatrix}$$

$$\tau = \mu \frac{\partial u}{\partial y}$$

MATRICE DI ROTAZIONE $[\bar{\bar{\Omega}}]$ → NO DEFORMAZIONE



$$\text{media} = \frac{\alpha_1 + (-\alpha_2)}{2} = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) dt \Rightarrow \boxed{\frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) = \frac{\frac{1}{2}(d\alpha_1 - d\alpha_2)}{dt}}$$

velocità media di rotazione

→ Se $d\alpha_1 = d\alpha_2$ avrò una rotazione rigida!

$$\bar{\bar{\Omega}} = \begin{bmatrix} 0 & \omega_z & \omega_y \\ -'' & 0 & \omega_x \\ -'' & -'' & 0 \end{bmatrix}$$ emisimmetrica

$$\bar{\omega} = \omega_x \vec{i} + \omega_y \vec{j} + \omega_z \vec{k}$$

Si nota come $\bar{\omega} = \frac{1}{2}\, rot(\vec{v})$

$$\begin{bmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ u & v & w \end{bmatrix} = \left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right)\vec{i} - \left(\frac{\partial w}{\partial x} - \frac{\partial u}{\partial z}\right)\vec{j} + \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)\vec{k}$$

FLUIDI DI STOKES

FLUIDI SENZA MEMORIA, LO SFORZO DIPENDE SOLO DALLA VELOCITÀ DI DEF.

$\underline{\underline{\phi}} = f(\underline{\underline{D}})$ matrici simmetriche

→ Si può dimostrare che esiste sempre un sist di riferimento per cui ho matrici diagonali.

$$\begin{bmatrix} \sigma_x & 0 & 0 \\ 0 & \sigma_y & 0 \\ 0 & 0 & \sigma_z \end{bmatrix} = f \begin{bmatrix} D_{xx} & 0 & 0 \\ 0 & D_{yy} & 0 \\ 0 & 0 & D_{zz} \end{bmatrix} \quad \text{inoltre} \quad \begin{bmatrix} \sigma_x & 0 & 0 \\ 0 & \sigma_y & 0 \\ 0 & 0 & \sigma_z \end{bmatrix} = \underbrace{\begin{bmatrix} p & 0 & 0 \\ 0 & p & 0 \\ 0 & 0 & p \end{bmatrix}}_{\text{PARTE STATICA VAR DI VOL}} + \underbrace{\begin{bmatrix} \sigma_x - p & 0 & 0 \\ 0 & \sigma_y - p & 0 \\ 0 & 0 & \sigma_z - p \end{bmatrix}}_{\text{PARTE DEVIATORICA DEFORMAZIONI}}$$

In un FLUIDO NEWTONIANO ABBIAMO UN LEGAME LINEARE TRA σ e D

$$\begin{cases} \sigma_x = p = \mu_{xx} D_{xx} + \mu_{xy} D_{yy} + \mu_{xz} D_{zz} \\ \sigma_y = p = \mu_{yx} D_{xx} + \mu_{yy} D_{yy} + \mu_{yz} D_{zz} \\ \sigma_z = p = \mu_{zx} D_{xx} + \mu_{zy} D_{yy} + \mu_{zz} D_{zz} \end{cases}$$

Si dimostra che $\mu_{ii} = a \quad \mu_{ij} = b$

$$\rightarrow \begin{cases} \sigma_x - p = a D_{xx} + b(D_{yy} + D_{zz}) \overbrace{+ bD_{xx} - bD_{xx}}^{\text{AGG E TOL}} \\ \sigma_y - p = a D_{yy} + b(D_{xx} + D_{zz}) + bD_{yy} - bD_{zz} \\ \sigma_z - p = a D_{zz} + b(D_{xx} + D_{yy}) + bD_{zz} - bD_{z} \end{cases} \Rightarrow \begin{cases} \sigma_x - p = (a-b) D_{xx} + b \operatorname{div}(\vec{v}) \\ \sigma_y - p = (a-b) D_{yy} + b \operatorname{div}(\vec{v}) \\ \sigma_z - p = (a-b) D_{zz} + b \operatorname{div}(\vec{v}) \end{cases}$$

sommando

$$\overbrace{\cancel{\sigma_x + \sigma_y} + \sigma_z}^{3p} - \cancel{3p} = (a-b)\operatorname{div}(\vec{v}) + 3b \operatorname{div}(\vec{v}) \rightarrow \operatorname{div}(\vec{v})\,(a-b+3b) = 0$$

- $\operatorname{div}(\vec{v}) = 0$ fluido incomp
- $a = 2b$

$$\boxed{\underline{\underline{\phi}} = -3b\, \underline{\underline{D}} + p\, \underline{\underline{I}} + b \operatorname{div}(\vec{v})\, \underline{\underline{I}}}$$

$$\underline{\underline{\phi}}_{xy} = \tau_z = -3b \frac{1}{2} \frac{d\gamma_t}{dt} + p + b \operatorname{div}(\vec{v})\ 0$$

$\underline{\underline{I}}$ → ha solo termini diag $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$$\tau_t = -\cancel{\mu}\, 3b \frac{1}{2} \frac{d\gamma_t}{dt}$$

$$\tau_t = -\mu \frac{d\gamma_t}{dt} \quad \rightarrow \quad 3b\frac{1}{2} = \mu \qquad b = \frac{2}{3}\mu$$

$$\boxed{\underline{\underline{\phi}} = -2\mu \underline{\underline{D}} \left(p + \frac{2}{3} \mu \operatorname{div}(\vec{v})\right) \underline{\underline{I}}}$$

EQUAZIONI

$$\begin{array}{l} 3\\ 1\\ 6\\ 1\\ \\ \end{array}\begin{cases} \rho(\vec{f}-\vec{a}) = \text{div}\,(\underline{\underline{\Phi}}) \\ \frac{d\rho}{dt}+\rho\,\text{div}\,(\vec{v})=0 \\ \underline{\underline{\Phi}} = \left(p+\frac{3}{2}\mu\,\text{div}\,(\vec{v})\right)\underline{\underline{I}} - 2\mu\,\underline{\underline{D}} \\ p=p(\rho) \\ \text{C.C.} \end{cases}$$

$$\xrightarrow[\text{lungo } x]{1 \text{ e } 3}$$

$$\rho(\vec{f}_x-\vec{a}_x) = \frac{\partial \Phi_x}{\partial x}+\frac{\partial \tau_z}{\partial y}+\frac{\partial \tau_y}{\partial z}$$

$$\begin{cases} \Phi_x = p + \frac{3}{2}\mu\,\text{div}(\vec{v}) - 2\mu\,\frac{\partial u}{\partial x} \\ \tau_z = -\chi\mu\,\frac{1}{2}\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right) \\ \tau_y = -\gamma\mu\,\frac{1}{2}\left(\frac{\partial u}{\partial z}+\frac{\partial w}{\partial x}\right) \end{cases}$$

SOSTITUISCO $\xrightarrow{\text{lungo } x}$ $\rho(f_x-a_x) = \frac{\partial p}{\partial x} + \frac{2}{3}\mu\,\frac{\text{div}(\vec{v})}{\partial x} - 2\mu\,\frac{\partial^2 u}{\partial x^2} - \mu\left(\frac{\partial^2 u}{\partial^2 y}+\frac{\partial v}{\partial x\partial y}+\frac{\partial^2 u}{\partial^2 z}+\frac{\partial w}{\partial x\partial y}\right)$

$$-\frac{1}{3}\mu\,\frac{\text{div}(\vec{v})}{\partial x} \qquad +\mu\,\frac{\text{div}(\vec{v})}{\partial x} \qquad \mu\nabla^2 u$$

$$\begin{cases} \rho(f_x-a_x) = \frac{\partial p}{\partial x} - \frac{1}{3}\mu\,\frac{\text{div}(\vec{v})}{\partial x} - \mu\,\overset{\text{div}(\text{grad}(u))}{\nabla^2 u} \\ \rho(f_y-a_y) = \frac{\partial p}{\partial y} - \frac{1}{3}\mu\,\frac{\text{div}(\vec{v})}{\partial y} - \mu\nabla^2 v \\ \rho(f_z-a_z) = \frac{\partial p}{\partial z} - \frac{1}{3}\mu\,\frac{\text{div}(\vec{v})}{\partial z} - \mu\nabla^2 z \end{cases} \Longrightarrow$$

LEGGE DEL MOTO

$$\boxed{\rho(\vec{f}-\vec{a}) = \text{grad}(p) - \frac{1}{3}\mu\,\text{grad}(\text{div}(\vec{v})) - \mu\nabla^2\vec{v}}$$

EQUAZIONE DI NEVIER-STOKES: è la legge del moto di un fluido newtoniano, incomprimibile al primo ordine

$$\begin{cases} \frac{d\rho}{dt}+\rho\,\text{div}(\vec{v})=0 \qquad \text{se } \rho=\text{cost} \quad \text{div}(\vec{v})=0 \\ \rho(\vec{f}-\vec{a}) = \text{grad}(p) - \cancel{\frac{1}{3}\mu\,\text{grad}}\overset{\text{div}=0}{\text{div}(\vec{v})} - \mu\nabla^2\vec{v} \\ \text{C.C.} \end{cases}$$

CONTINUITÀ 1

3

$\frac{n}{4}$ (u, v, w, p)

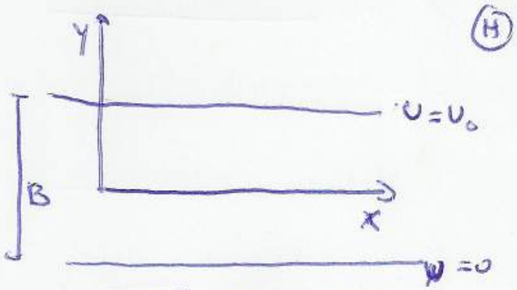
$$\rho(\vec{f}-\vec{a}) = \text{grad}\,p - \mu\nabla^2\vec{v} \qquad \vec{a} = -\frac{1}{\rho}\,\text{grad}(p) + \frac{\mu}{\rho}\nabla^2\vec{v}+\vec{f} \quad \text{ma} \quad \vec{a} = \frac{\partial\vec{v}}{\partial t}+\frac{\partial\vec{v}}{\partial x}\overset{u}{\widehat{\frac{dx}{dt}}}+\frac{\partial\vec{v}}{\partial y}v+\frac{\partial\vec{v}}{\partial z}w$$

$$\frac{\partial\vec{v}}{\partial t}+\frac{\partial\vec{v}}{\partial x}u+\frac{\partial\vec{v}}{\partial y}v+\frac{\partial\vec{v}}{\partial z}w = -\frac{1}{\rho}\,\text{grad}(p) + \frac{\mu}{\rho}\,\text{div}\,(\text{grad}\,\vec{v}) + \vec{f}$$

$$\begin{array}{l} x\\ \\ y\\ \\ z\\ \\ \\ \end{array}\begin{cases} \frac{\partial u}{\partial t} + \overbrace{\frac{\partial u}{\partial x}u + \frac{\partial u}{\partial y}v + \frac{\partial u}{\partial z}w}^{\text{non lineari}} = -\frac{1}{\rho}\,\frac{\partial p}{\partial x} + \frac{\mu}{\rho}\left(\frac{\partial^2 u}{\partial^2 x}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}\right) + f_x \\ \frac{\partial v}{\partial t} + \frac{\partial v}{\partial x}u + \frac{\partial v}{\partial y}v + \frac{\partial w}{\partial z}w = -\frac{1}{\rho}\,\frac{\partial p}{\partial y} + \frac{\mu}{\rho}\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}+\frac{\partial^2 v}{\partial z^2}\right) + f_y \\ \frac{\partial w}{\partial t} + \frac{\partial v}{\partial x}u + \frac{\partial w}{\partial y}v + \frac{\partial w}{\partial z}w = -\frac{1}{\rho}\,\frac{\partial p}{\partial z} + \frac{\mu}{\rho}\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}+\frac{\partial^2 w}{\partial z^2}\right) + f_z \\ \text{div}(\vec{v})=0 \rightarrow \frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0 \end{cases}$$

MOTO LAMINARE:

1) LASTRA SCORREVOLE; ~~e~~ grad(p) dato e cost



Ⓗ Newtoniano, incomprimibile, moto staz., $\bar{F}=0$, $v=0$

Scrivo Navier-Stokes lungo la x e y + cont

$$\vec{a} = -\frac{1}{\rho}\,\text{grad}(p) + \frac{\mu}{\rho}\,\bar{\nabla}^2(\bar{v})$$

$$\begin{cases} x: \ \overset{HP\,SS}{\cancel{\frac{\partial u}{\partial t}}} + \overset{③}{\cancel{\frac{\partial u}{\partial x}u}} + \overset{v=0}{\cancel{\frac{\partial u}{\partial y}v}} + \overset{HP}{\cancel{\frac{\partial u}{\partial z}w}} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \frac{\mu}{\rho}\cancel{\nabla^2}\left(\overset{③}{\cancel{\frac{\partial^2 u}{\partial x^2}}} + \frac{\partial^2 u}{\partial y^2} + \overset{HP}{\cancel{\frac{\partial^2 u}{\partial z^2}}}\right) \overset{HP}{\cancel{+f_x}} \\ y: \ \cancel{\frac{\partial v}{\partial t}} + \overset{v=0}{\cancel{\frac{\partial v}{\partial x}u}} + \cancel{\frac{\partial v}{\partial y}v} + \cancel{\frac{\partial v}{\partial z}w} = \frac{1}{\rho}\frac{\partial p}{\partial y} + \frac{\mu}{\rho}\left(\overset{v=0}{\cancel{\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2}}}\right) = \\ \text{CONT}: \ \frac{\partial u}{\partial x} + \overset{0}{\cancel{\frac{\partial v}{\partial y}}} = 0 \quad \Rightarrow \frac{\partial u}{\partial x} = 0 \ ③ \end{cases}$$

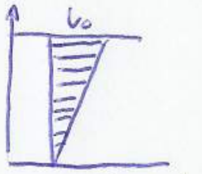
$$\begin{cases} \frac{\partial u}{\partial x} = 0 \rightarrow \text{la velocità non varia lungo x} \\ \frac{1}{\rho}\frac{\partial p}{\partial y} = 0 \rightarrow \text{la pressione non varia lungo y} \\ \frac{\mu}{\cancel{\rho}}\frac{\partial^2 u}{\partial y^2} = \frac{1}{\cancel{\rho}}\frac{\partial p}{\partial x} \end{cases}$$

→ noto che p è f di x e u è f di y ⇒ derivate tot.

Ⓗ $p = +\infty$ e $p = -\infty$ → $\frac{\partial p}{\partial x} = \text{cost}$

$$\mu\frac{d^2u}{dy} = \frac{dp}{dx} \xrightarrow{\text{integro}} \cancel{\mu}\frac{du}{dy} = \frac{\cancel{C_0}\,C_1}{\mu} \xrightarrow{\text{integro}} u = C_1 y + C_2$$

$$\begin{cases} u\left(\frac{B}{2}\right) = U_0 & C_1\frac{U_0}{B} \\ u\left(\frac{B}{2}\right) = 0 & C_2 = \frac{U_0}{2} \end{cases}$$

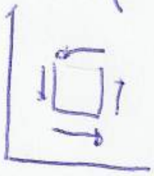


$$u = U_0\left(\frac{y}{B} + \frac{1}{2}\right)$$

SFORZO

$$\underline{\underline{\Phi}} = p\,\underline{\underline{I}} \cancel{+} -2\mu\,\underline{\underline{D}} = \begin{bmatrix} p & -\mu\frac{U_0}{B} \\ -\mu\frac{U_0}{B} & p \end{bmatrix}$$

$$T = -2\mu\frac{1}{2}\left(\frac{\partial u}{\partial y} + \cancel{\frac{\partial v}{\partial x}}\right) \rightarrow \boxed{\overset{\text{NEWTON}}{\tau = -\mu\frac{U_0}{B}}}$$



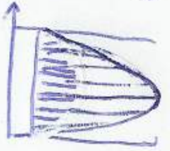
2) LASTRE ferme, grad p dato
come prima

$$\mu\frac{du^2}{dy^2} = \frac{dp}{dx} \qquad \text{noto che} \quad \frac{\partial p}{\partial x} \overset{\text{div } \gamma}{\leq} \gamma \cdot \overset{t=\text{cost}}{\overline{\frac{\partial}{\partial x}\left(\frac{p}{\gamma}\right)}} = +\gamma - J$$

$$\cancel{\mu}\frac{du^2}{dy^2} = -\frac{\gamma J}{\mu} \xrightarrow{\text{integro}} \frac{du}{dy} = -\frac{\cancel{\gamma} J}{\mu}y + C_1 \xrightarrow{\text{integro}} U = -\frac{\gamma J}{\mu}\frac{y^2}{2} + C_1 y + C_2$$

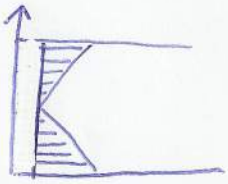
$$\begin{cases} U\left(\frac{B}{2}\right) = 0 \rightarrow C_1 = 0 \\ U\left(-\frac{B}{2}\right) = 0 \rightarrow C_2 \frac{\gamma J}{2\mu}\frac{B^2}{4} \end{cases}$$

$$U = \frac{\gamma J}{2\mu}\left(\frac{B^2}{4} - y^2\right)$$



SFORZO

$$\underline{\Phi} = p\,\underline{\underline{\Phi}} - 2\mu\,\underline{\underline{D}} = \begin{bmatrix} p & \gamma J y \\ \gamma J y & p \end{bmatrix}$$



$$T = 0 - 2\mu\frac{1}{2}\left(\frac{\partial u}{\partial y} + \cancel{\frac{\partial v}{\partial x}}\right) = \gamma J y$$

ⓃⒷ è curioso che p non è f di μ lo sforzo

3) MOTO LAM: LAM MOVIMENTO + GRAD DI PRESSIONE

→ USO LA SOVRAPPOSIZIONE DEGLI EFFETTI

MOTO IN TUBAZIONI CIRCOLARI CILINDRICHE

HP : STAZ $F=-g\nabla z$ $V_m=v=0$

$$\begin{cases} 3 & \vec{a} = \frac{1}{\rho}\,\text{grad}(p) + \frac{\mu}{\rho}\nabla^2(\vec{v}) + \vec{F} \\ 1 & \text{div}(\vec{v}) = 0 \end{cases} \quad 4\ \text{eq}$$

CON VARIE semplificazioni banali dovute alle ipotesi

$$\begin{cases} -\frac{1}{\rho}\frac{\partial p}{\partial x} + \frac{\mu}{\rho}\left(\frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right) - g\frac{\partial z}{\partial x} = 0 \\ -\frac{1}{\rho}\frac{\partial p}{\partial y} - g\frac{\partial z}{\partial y} = 0 \qquad \frac{\partial}{\partial y}\left(\frac{p}{\gamma}+z\right) = 0 \\ -\frac{1}{\rho}\frac{\partial p}{\partial n} - g\frac{\partial z}{\partial n} = 0 \rightarrow \frac{\partial}{\partial n}\left(\frac{p}{\gamma}+z\right) = 0 \end{cases}$$

→ la distribuzione idrostatica lungo le normali

$\frac{\partial u}{\partial x} = 0$ → lungo x non varia la velocità

$$\frac{\mu}{\rho}\left(\frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right) = g\underbrace{\frac{\partial}{\partial \cancel{x}}\left(\frac{p}{\gamma}+z\right)}_{-J} \rightarrow \frac{\partial u^2}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = -\frac{\cancel{\mu}\gamma}{\mu}J \xrightarrow[\text{POLARI}]{\text{IN COOR}} = \frac{1}{R}\frac{\partial}{\partial R}\left(R\frac{\partial u}{\partial R}\right) \overset{?}{+} \cancel{\frac{1}{R^2}\frac{\partial^2 u}{\partial \theta^2}} = -\frac{\gamma}{\mu}J$$

simmetr

$$\rightarrow \frac{\partial}{\partial R}\left(R\frac{\partial u}{\partial R}\right) = -\frac{\gamma J R}{\mu} \xrightarrow{\text{INTEGRO}} R\frac{du}{dR} = -\frac{\gamma J}{\mu}\frac{R^2}{2} + c_1 \rightarrow$$

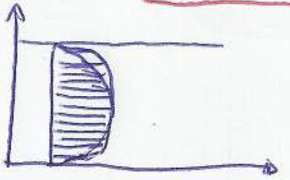
AL centro $(R=0)$ $V=max \Rightarrow \frac{du}{dR}=0$ $R=0$ ⇓ $C_1=0$

$$\xrightarrow{\text{integro}} \boxed{U = \frac{-\gamma J}{2\mu}\frac{R^2}{2} + C_2}$$

→ IN $R=R_0$ $U=0$ COND DI ADERENZA

$$C_2 = \frac{\gamma J}{4\mu}R_0^2$$

$$\boxed{v = \frac{\gamma J}{4\mu}\left(R_0^2 - R^2\right)}$$



$$V_m = \frac{1}{A}\int_A u\,dA = \frac{u_{max}}{2} = \frac{\gamma J}{4\mu}\frac{R_0^2}{2} = \frac{\gamma J}{4\mu}\frac{D^2}{4}$$

$$V_m = \frac{\gamma J}{4\mu}\frac{D^2}{4} \rightarrow J = \frac{32\mu}{\gamma D}\frac{V_m}{D}$$ <u>LEGAME LINEARE</u>

$$\rightarrow J = \frac{32\mu\cdot 2}{\rho g D\cdot V_m}\frac{V_m\cdot V_m}{D\,2} \rightarrow J = \frac{64}{Re}\frac{V_m^2}{2gD}$$

$\frac{1}{Re}$

→ il legame con V_m è ancora lineare solo che esprimendolo in termine cinetico c'è V_m^2 ma un V_m va via con Re

SFORZO

$\underline{\underline{\Phi}} = p\underline{\underline{I}} - 2\mu\underline{\underline{D}}$

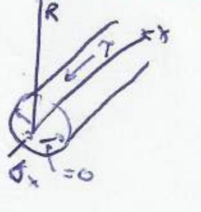
R non varia lungo x R=cost

$\tau_{Rx} = -\cancel{2}\mu\frac{1}{\cancel{2}}\left(\frac{\partial u}{\partial R} + \cancel{\frac{\partial R}{\partial x}}\right)$

$\tau_{Rx} = \cancel{\mu}\,\frac{-\gamma J}{2\cancel{\mu}}R \Rightarrow \tau = \frac{-\gamma J R}{2}$

$\sigma_x = p$ $\sigma_R = p$

PARETE LATERALE



LUNGO IL PIANO yn FACCIA

$$\tau_{xy} = -2\mu D_{xy} = -\cancel{2}\mu\frac{1}{\cancel{2}}\left(\frac{\partial u}{\partial y} + \overset{=0}{\cancel{\frac{\partial v}{\partial x}}}\right) = -\mu\left(\frac{du}{dR}\frac{\partial R}{\partial y}\right)$$

$\frac{du}{dR} = \frac{-\gamma J}{4\mu}2R$ $R = \sqrt{x^2+n^2}$ $\cancel{2}R\frac{\partial R}{\partial y} = \cancel{2}y$ $\frac{\partial R}{\partial y} = \frac{y}{R}$

$$-\mu\frac{du}{dR}\frac{\partial R}{\partial y} = -\cancel{\mu}\,\frac{-\gamma J}{4\cancel{\mu}}2\cancel{R}\frac{y}{\cancel{R}} = \frac{\gamma J y}{2}$$ (Pr)

ANALOGO τ_{xn} (*)

$$\tau = \sqrt{\tau_{xy}^2 + \tau_{xn}^2} = \frac{\gamma R J}{2} \quad (*)$$

SI vede che le τ sono opposte, lo stato di sforzo <u>NON</u> è nullo ma solo l'azione totale

$$T_x = \frac{1}{A_x}\int_{A_x}\tau\,dA_x = 0$$

$$\Pi_\mu = \int_A \mu\frac{\partial \bar{u}}{\partial n}dA$$

$$T_R = \int_{A_R}\phi_{Rx}\,dA_R = \frac{\gamma J R_0}{2}2R_0\pi L = \boxed{\gamma J W}$$ SFORZO SULLA PARETE LATERALE

MOTI TURBOLENTI

OK

Le equazioni di Navier-Stokes non sono facilmente risolvibili nel continuo, dunque si rende il campo discreto, creando una griglia, e trovando le sol nei punti.
Si moltiplicano variabili ed equazioni ma si trova una sol.

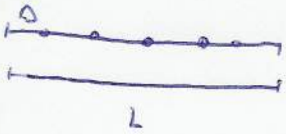
Sovrapposto ⟹ OK

- La scelta del passo della griglia e acc quando $\frac{du}{dx} = \lim_{\Delta x \to 0} \frac{\Delta u}{\Delta x}$

più Δx è piccolo + aumenta la precisione

La griglia dipende dal grado della turbolenza, ma questo lo conosco dopo aver trovato le sol

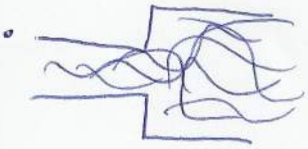
macro vortici $= \overset{\text{SCALA}}{L}$ vortici intermedi $= \overset{\text{SCALA}}{\ell}$ microvortici $= \overset{\text{SCALA}}{\lambda}$



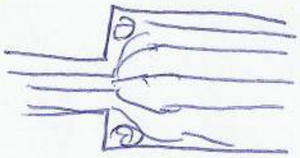
$$\begin{cases} N\Delta > L \\ \Delta < \lambda \end{cases} \quad N > \frac{L}{\lambda} \quad N > Re^{\frac{3}{4}} \quad \text{IN 3D} \quad Re^{\frac{9}{4}}$$

NUMERO MINIMO DI PASSI

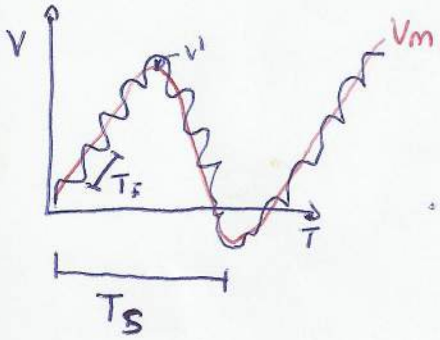
NUMERO MINIMO DI PUNTI



NON HO INTERESSE A CONOSCERE I VALORI ISTANTANEI, PERCIÒ QUELLI NON ⟹



STUDIAMO LA velocità avente una periodicità T_S che contiene un'altra periodicità T_F



DOVENDO STUDIARE IL PROBLEMA IN UN INTERVALLO DI Tempo $T_F < T < T_S$

⟶ RISCRIVO OGNI GRANDEZZA COME $\bar{V} = \bar{V}_m + \bar{V}'$

termine medio

Fluttuazione

$$\bar{V}_m = \frac{1}{T}\int_0^T \bar{V}\, dt$$

$$V_m = \frac{1}{T}\int_0^T V_m + v' \, dt = \frac{1}{T}\int_0^T V_m + \frac{1}{T}\int_0^T v' \, dt \Rightarrow \cancel{V_m} = \frac{1}{\cancel{T}}\cancel{V_m}\cancel{T} + \frac{1}{T}\int_0^T v' dt \Rightarrow \boxed{\frac{1}{T}\int_0^T v' \, dt = 0}$$

FLUTTUAZIONE A MEDIA NULLA

NAVIER-STOKES

ASSE x ⟶ $$\overset{\textcircled{1}}{\frac{\partial u}{\partial t}} + \overbrace{u\frac{\partial u}{\partial x} + v\frac{\partial v}{\partial y}, w\frac{\partial w}{\partial z}}^{\textcircled{2}} = \overbrace{\frac{\mu}{\rho}\nabla^2 u}^{\textcircled{3}} - \frac{1}{\rho}\frac{\partial p}{\partial x} + f_x$$

① $$\left(\frac{\partial u}{\partial t}\right)_m = \frac{1}{T}\int_0^T \frac{\partial u}{\partial t}\, dt = \frac{\partial}{\partial t}\left[\frac{1}{T}\int_0^T u_m\, dt\right] = \frac{\partial u_m}{\partial t}$$

② $$\left(\frac{\partial u}{\partial x}u\right)_m = \left((u_m+u')\frac{\partial(u_m+u')}{\partial x}\right)_m = \left(u_m\frac{\partial u_m}{\partial x}\right)_m + \cancel{\left(u_m\frac{\partial u'}{\partial x}\right)_m} + \cancel{\left(u'\frac{\partial u_m}{\partial x}\right)_m} + \left(u'\frac{\partial u'}{\partial x}\right)_m = u_m\frac{\partial u_m}{\partial x} + \frac{1}{T}\int_0^T u'\frac{\partial u'}{\partial x}$$

⟶ FLUTTUAZ NULLA

IL PRODOTTO DI 2 FLUTT ≠ 0

③ $$(\nabla^2 u)_m = \frac{1}{T}\int_0^T \nabla^2 u\, dt = \nabla^2\left[\frac{1}{T}\int_0^T u\, dt\right] = \nabla^2 u_m$$

$$\frac{\partial u_m}{\partial t} + u_m \frac{\partial u_m}{\partial x} + \underbrace{\frac{1}{T}\int u' \frac{\partial u'}{\partial x}\, dt}_{} + v_m \frac{\partial u_m}{\partial y} + \underbrace{\frac{1}{T}\int v' \frac{\partial u'}{\partial y}\, dt}_{} + w_m \frac{\partial u_m}{\partial z} + \underbrace{\frac{1}{T}\int w' \frac{\partial u'}{\partial z}\, dt}_{} = \frac{\mu}{\rho}\nabla^2 u_m + \frac{1}{\rho}\frac{\partial P_m}{\partial z} + f_{xm}$$

$\frac{1}{T}\int \left(u' \frac{\partial u'}{\partial x} + v' \frac{\partial u'}{\partial y} + w' \frac{\partial u'}{\partial z} \right) dt$ noto che $\frac{\partial (u'u')}{2} = u' \frac{\partial u'}{\partial x} + u' \frac{\partial u'}{\partial x}$

$$\to \frac{1}{T}\int \partial\, \frac{\partial (u'u')}{\partial x} - \underbrace{\cancel{\frac{\partial u'}{\partial x}}}_{} u' + \frac{\partial (u'v')}{\partial y} - \underbrace{u' \cancel{\left(\frac{\partial v'}{\partial y}\right)}}_{} + \frac{\partial (w'u')}{\partial z} - \underbrace{\partial \cancel{\left(\frac{w'}{\partial z}\right)} u'}_{}$$

SPARISCONO con l'eq di CONT $\begin{cases} \frac{\partial u_m}{\partial x} + \frac{\partial u_m}{\partial y} + \frac{\partial u_m}{\partial z} = 0 \\ \frac{\partial u'}{\partial x} + \frac{\partial u'}{\partial y} + \frac{\partial u'}{\partial z} = 0 \end{cases} \Rightarrow \frac{\partial (u_m u_i)}{\partial x} + (\dots) = 0$

$$\Rightarrow \frac{\partial u_m}{\partial t} + \frac{u_m}{\partial x} u_m + \frac{\partial v_m}{\partial y} v_m + \frac{\partial w_m}{\partial z} w_m + \frac{1}{T}\frac{\partial}{\partial x}\int_0^T u'u' + \frac{1}{T}\frac{\partial}{\partial y}\int_0^T v'u' + \frac{1}{T}\frac{\partial}{\partial z}\int_0^T w'u' = \frac{1}{\rho}\frac{\partial P_m}{\partial x} + \nabla^2 u_m + f_{xm}$$

∠ EQUAZIONE DI REYNOLDS

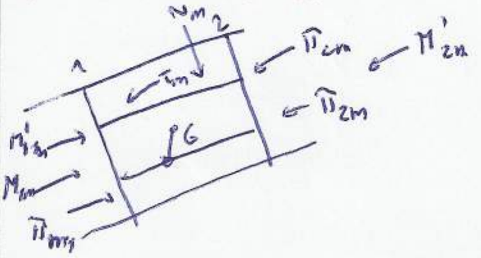
SFORZO

$G + \Pi_p + \Pi_\mu + M + I$

← flusso di quantità di moto dovuto alle perturbazioni

$\overline{G_n} + \overline{\Pi_{pn}} + \Pi_{\mu n} + \overline{\Pi} + M'_n + I_m = 0$

MOTO MEDIO STAZ.



$M'_{1m} = \frac{1}{T}\int (u_1 v)_m \, dA$ $u_1 v_1$ in $A_1 \neq u_1 v_1$ in A_2

↓

ma la loro media è uguale

$M'_{1n} = M^{2}_{1n}$ ⇒ si ANNULLANO

$T = \gamma W J_m$

$T_{mx} = -\gamma W J_m = T_\mu + M'_{mx}$

$T_{mx} = T_{\mu x} + M'_{mx}$

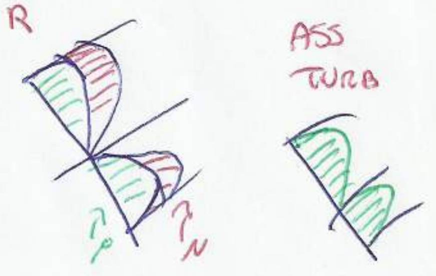
$\tau_{mx} = \frac{T_{mx}}{2\pi r L} = \frac{-\gamma \cancel{\pi} r^{\cancel{2}} \cancel{L} J_m}{2\cancel{\pi} \cancel{r} \cancel{L}} = -\frac{\gamma r J_m}{2} = \tau_{\mu x} + \tau_p$

$\tau_{\mu x} = +2\mu D_{rx} = \mu \frac{du_m}{dr}$ ← viscoso → Newton

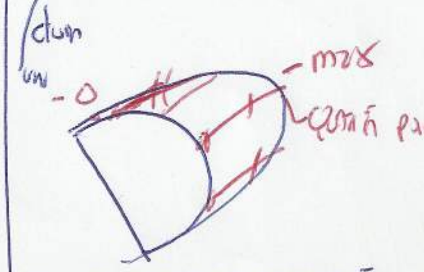
$\tau_p = \frac{M'_n}{sup} = \rho \int_{A_L} (u' v'_r)_n \, dA_L$ ← $\frac{M'_n}{sup}$

$$\boxed{\tau = \mu \frac{du_m}{dr} + \rho \int_{A_L} (u' v')_n \, dA_L = \gamma \frac{R J_m}{2}}$$

STATO DI SFORZO AL VARIARE DI R

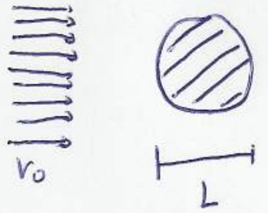


PROFILO VELOCITÀ



IL PROFILO TURBOLENTO È uguale a quello viscoso con meno una quantità max al centro

EQ DI EULERO E STOKES $\cancel{a}$ - $\frac{\partial \bar v}{\partial t} + u\frac{\partial \bar v}{\partial x} + v\frac{\partial \bar v}{\partial y} + w\frac{\partial \bar v}{\partial t} = -\frac{1}{\rho}\,\text{grad}\,p + \frac{\mu}{\rho}\nabla^2\bar v + \vec f$



Voglio studiare il moto nel caso di fluido newtoniano, incompr. pesante al variare di $L, \bar v_0, \rho, \mu$

$\Rightarrow 10^9$ simulazioni minime → semplifico

1) MOTO STAZIONARIO $\frac{\partial \bar v}{\partial t} = 0$

2) ADIMENSIONALIZZO → (ρ, V_0, L)

① $\bar V^* = \frac{\vec V}{\bar V_0}$ → $U^* = \frac{U}{U_0}$ $V^* = \frac{V}{V_0}$ $W^* = \frac{W}{W_0}$

② $X^* = \frac{X}{L}$

4

③ $-\frac{1}{\rho}\,\text{grad}(p) + f = -\frac{1}{\rho}\,\text{grad}\,p - g\,\text{grad}\,z = -\frac{1}{\rho}\nabla(p+\gamma z)$

$\tilde p = p + \gamma z \leq \left[\frac{N}{m^2}\right]$ è una pressione $\quad \overset{\|}{-\frac{1}{\rho}\nabla(\tilde p)}$

$\tilde p^* = \frac{\tilde p}{\rho V_0^2}$

$$\rightarrow \cancel{\frac{\partial V}{\partial t}} + U^* V_0 \frac{\partial (V^* V_0)}{\partial x^* L} + V^* V_0 \frac{\partial V^* V_0}{\partial y^* L} + W^* V_0 \overset{\frac{V_0^2}{L}}{\frac{\partial V V_0}{\partial (z^* L)}} \Big) = -\frac{1}{\rho}\frac{\rho}{\cancel{L}}\cancel{V_0^2}\nabla^*(\tilde p^*) + \overset{\frac{1}{Re}}{\frac{\mu}{\rho}\frac{1}{L^2}\frac{\cancel{V_0}}{V_0}}$$

$$\left[U^*\frac{\partial U^*}{\partial x^*} + V^*\frac{\partial V^*}{\partial y^*} + W^*\frac{\partial V^*}{\partial z^*}\right] = -\nabla^*\tilde p^* + \frac{1}{Re}\nabla^{*2}\bar V^*$$

$$\boxed{\nabla^*\tilde p^* = \frac{1}{Re}\nabla^{*2}\vec v^* - a_c^*}$$

Re → 0 → conta $a_c\,\nabla^{*2}\vec V$

Re → ∞ → conta $\cancel{\frac{1}{R}}\,\vec a_c$

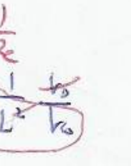
Re → 0

$$\nabla\tilde p^* = \frac{1}{Re}\nabla^{*2}\vec V^* \Rightarrow \frac{L}{\cancel{\rho} V_0^{\cancel{2}}}\nabla\tilde p = \frac{1}{\cancel{\rho}\frac{V_0 L}{\mu}}\frac{\cancel{L^2}}{\cancel{V_0}}\nabla^2\vec V \Rightarrow \boxed{\nabla\tilde p = \mu\nabla^2\vec V}$$

LEGGE DI STOKES per Re → 0

→ Trovo che $p = f(\mu, L, V_0)$ per Re → 0

$\Pi_F = \frac{F}{\underbrace{\mu\frac{V_0}{\cancel{L}}}_{\text{press}}\underbrace{L^2}_{\text{area}}} = \text{cost}$ → se ho Re bassi sento ovunque l'effetto di μ solo all'infinito la velocità torna w



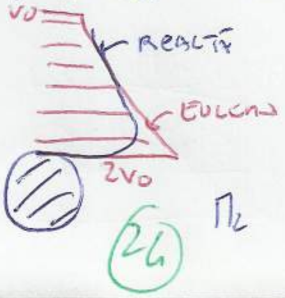
Re → ∞

$\nabla^*\tilde p^* = -\vec a_c^*$ → $\frac{\cancel{L}}{\rho V_0^2}\nabla\tilde p = -\frac{\cancel{L}}{V_0^2}\vec a_c$ $\quad \nabla\tilde p = -\rho\,\vec a_c \Rightarrow \nabla(p\gamma + z) = -\rho\,\vec a_c$

$\nabla p = -\rho(-g\nabla z - a_c)$

$$\boxed{\nabla p = -\rho(\vec F - a_c)}$$

EQ DI EULERO



⇒ L'eq di Eulero non prende in esame μ quindi per la continuità la velocità raddoppia quando V ↑ e ho Re ↑ la EULERO funziona

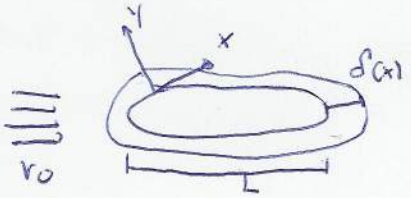
Ma vicino all'oggetto per l'aderenza ho v bassi e Re ↓ ⇒ STRATO LIMITE

ZONA DI STRATO LIMITE → Lo strato dove la velocità è compresa tra 0 e il 99%

E ciò che unisce l'equazione di Navier-Stokes (μ) all'equazione di Eulero.

L'equazione di Eulero funziona all'esterno, di moto non viscoso o non turbolento. (STRATO LIMITE COMPLETAMENTE SVILUPPATO)

STRATO COMPLETAMENTE SVILUPPATO ⇒ $Re \to +\infty$



$$U^* = \frac{U}{v_0} \quad V^* = \frac{V}{v_0}$$

Eq CONT: $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$

$$\frac{\cancel{v_0}}{\cancel{L}} \frac{\partial u^*}{\partial x^*} + \frac{\cancel{v_0}}{\cancel{L}} \frac{\partial v^*}{\partial y^*} = 0$$

$$\begin{cases} \nabla^* \tilde{p}^* = \frac{\nabla^{*2} \vec{v}}{Re} - \left(u^* \frac{\partial \vec{v}^*}{\partial x^*} + v^* \frac{\partial \vec{v}}{\partial y^*} \right) \\ \frac{\partial u^*}{\partial x^*} + \frac{\partial v^*}{\partial y^*} = 0 \end{cases}$$

→ lungo x $\quad \frac{\partial \tilde{p}^*}{\partial x} = \frac{1}{Re} \left(\frac{\partial^2 u^*}{\partial x^{*2}} + \frac{\partial^{*2} u^*}{\partial y^{*2}} \right) - \left(u^* \frac{\partial^* u^*}{\partial x^*} + v^* \frac{\partial u^*}{\partial y^*} \right)$

$$x^* = \frac{x}{L} = \Theta(1) \quad y^* = \frac{\overset{\nearrow \text{ord } \delta}{Y}}{L} = \Theta(\varepsilon) \quad U^* = \frac{U}{V_0} = \Theta(1) \qquad \varepsilon \doteq \frac{\delta}{L} \ll 1 \ \nearrow \text{elevato } Re$$

CONT

$$\frac{\partial u^*}{\partial x^*} + \frac{\partial v^*}{\partial y^*} \Rightarrow \frac{\Theta(1)}{\Theta(1)} + \frac{\frown}{\Theta(\varepsilon)} \quad V^* = \Theta(\varepsilon)$$

NAVIER STOKES

$$\frac{\partial^* u^{*2}}{\partial x^{*2}} + \frac{\partial^2 u^*}{\partial y^{*2}} \Rightarrow \underset{\searrow \text{TRASCURABILE}}{\cancel{\Theta(1)}} + \frac{\Theta(1)}{\Theta(\varepsilon^2)}$$

→ Ho massima diffusione lungo la y

$$u^* \frac{\partial^* u^*}{\partial x^*} + v^* \frac{\partial u^*}{\partial y^*} \Rightarrow \Theta(1) + \cancel{\Theta(\varepsilon)} \frac{\Theta(1)}{\cancel{\Theta(\varepsilon)}}$$

→ Poiché vanno sommate e devono avere le stesse dimensioni

$$\Rightarrow \frac{1}{Re} = \Theta(\varepsilon^2) \Rightarrow Re \to +\infty$$

$$\frac{1}{Re} \propto \varepsilon^2 \to \frac{1}{Re} \propto \left(\frac{\delta}{L} \right)^2 \to \frac{1}{\sqrt{Re}} \propto \frac{\delta}{L}$$

$$\Rightarrow \text{LUNGO } x \begin{cases} \frac{\partial \tilde{p}^*}{\partial x} = \frac{1}{Re} \frac{\partial^2 u^*}{\partial y^{*2}} - \left(u^* \frac{\partial u^*}{\partial x^*} + v^* \frac{\partial u^*}{\partial y^*} \right) \\ \frac{\partial u^*}{\partial x^*} + \frac{\partial v^*}{\partial y^*} = 0 \end{cases} \to \frac{\partial \tilde{p}^*}{\partial x} = \Theta(1)$$

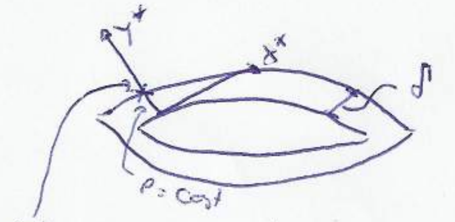
$$\Rightarrow \text{LUNGO } y \quad \frac{\partial \tilde{p}^*}{\partial y} = \frac{1}{Re} \left(\overset{\text{TRAS}}{\cancel{\frac{\partial^2 v^*}{\partial x^{*2}}}} + \frac{\partial^2 v^*}{\partial y^{*2}} \right) - \left(u^* \frac{\partial v^*}{\partial x^*} + v^* \frac{\partial v^*}{\partial y^*} \right)$$

$$\theta(\varepsilon^2) \left(\Theta(\varepsilon) + \frac{\cancel{\Theta(\varepsilon)}}{\cancel{\Theta(\varepsilon^2)}} - \left(\Theta(1)\, \Theta(\varepsilon) + \Theta(\varepsilon) \frac{\cancel{\Theta(\varepsilon)}}{\cancel{\Theta(\varepsilon)}} \right) \right) = \Theta(\varepsilon)$$

LUNGO y il gradiente di pressione è trascurabile!!

EQ INDEFINITA DI STRATO LIMITE

$$\begin{cases} \frac{\partial \tilde{p}^*}{\partial x^*} = \frac{1}{Re}\frac{\partial^2 u^*}{\partial y^*} - \left(u^* \frac{\partial^* u^*}{\partial x^*} + v^* \frac{\partial u^*}{\partial y^*}\right) \\ \frac{\partial \tilde{p}^*}{\partial y^*} = 0 \quad \text{(NELLO STRATO LIMITE } \tilde{p} = \text{cost} \\ \frac{\partial u^*}{\partial x^*} + \frac{\partial v^*}{\partial y^*} = 0 \end{cases}$$



eulero → bernulli → perdo gli effetti della viscosità

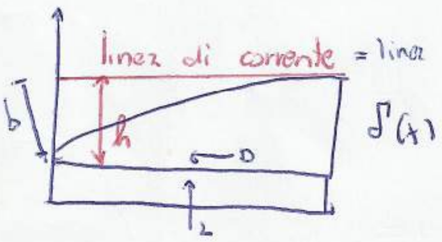
$$z + \frac{p}{\gamma} + \frac{V^2}{2g} = \text{cost} \rightarrow z\gamma + p + \frac{V^2}{2}\rho = c_1 \rightarrow \tilde{p} + \frac{V^2}{2}\rho = c_1 \Rightarrow \frac{\partial}{\partial x^*}\left(\tilde{p} + \frac{V^2}{2}\rho\right) = 0$$

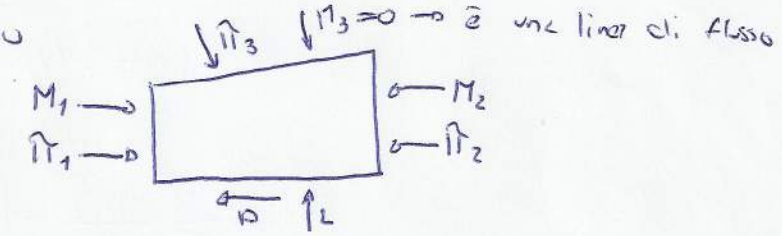
→ ADIMENSIONALIZZO QUELLO INTERNO $\frac{\partial}{\partial x^*}\left(\frac{\tilde{p}}{\rho V_0} + \frac{\cancel{\rho}}{\cancel{\rho} V_0^2}\frac{V^2}{2}\right) = 0 \rightarrow \frac{\partial}{\partial x^*}\left(\tilde{p}^* + \frac{V^{*2}}{2}\right) = 0$

$$\frac{\partial}{\partial x^*}\tilde{p}^* = -\frac{1}{2}\frac{\partial}{\partial x^*}\left(V^{*2}\right) = V^* \boxed{\frac{\partial V^*}{\partial x^*} = \frac{\partial}{\partial x^*}(\tilde{p}^*)}$$

→ Eq governata solo dalle cond di monte ma non da quelle di valle!!

CALCOLO DEL DRAG CON EQUILIBRIO LOCALE





lungo x : $|\vec{M}_1| + |\vec{\Pi}_1| + \Pi_3^x = |\vec{\Pi}_2| + |\vec{M}_2| + |\vec{D}|$

nella zona di strato limite $p = \text{cost}$

$\Pi_1 = p_0 h b = p_0 A_1$

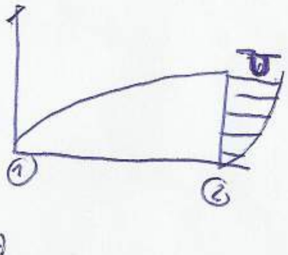
$\Pi_2 = p_0 \delta(x) b = p_0 A_2$

$\Pi_3 = p_0 A_3 = \cancel{p_0 (A_2 - A_1)}$

$\Pi_1 \mp \Pi_2 + \Pi_3 = p_0 (A_1 - A_2 + A_2 - A_1) = 0$

$\Pi_{3x} = p_0 A_3^x = p_0 (A_2 - A_1)$

$\vec{M} = \rho \int_A \vec{V} V_m \, dA \qquad |M_1| = \rho U U h b \qquad |M_2| = \rho b \int_0^\delta u^2 dy$



Poiché $Q_1 = Q_2 \rightarrow \cancel{b} h U = \cancel{b}\int_0^\delta u \, dy$

$|M_1| = \rho b U \int_0^\delta u \, dy$

$|M_2| = \rho b \int_0^\delta u^2 dy$

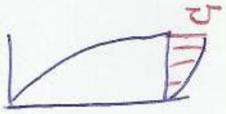
$$|\vec{D}| = |M_1| - |M_2| = \rho b U^2 \overbrace{\int_0^\delta \frac{u}{U}\left(1 - \frac{u}{U}\right) dy}^{\theta}$$

$$\boxed{\vec{D} = \rho b U^2 \theta}$$

$$\frac{d|\vec{D}|}{dx} = b\tau \rightarrow \cancel{b}\tau = \cancel{b}\rho\, U^2 \frac{\theta}{dx} \qquad \boxed{\tau = \rho\, U^2 \frac{\theta}{dx}}$$

ADIMENSIONALIZZAZIONE

$$\frac{u}{U} = f\left(\frac{y}{\delta}\right) \rightarrow \text{2 CONDIZIONI AL CONTORNO} \begin{cases} f\left(\frac{y}{\delta}=0\right)=0 \\ f\left(\frac{y}{\delta}=\delta\right)=1 \rightarrow \text{in } \delta \text{ ho proprio } U \end{cases}$$

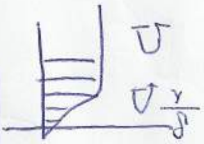


$$\theta = \int_0^\delta \frac{u}{U}\left(1-\frac{u}{U}\right) dy \rightarrow \int_0^\delta f\left(\frac{y}{\delta}\right)\left[1-f\left(\frac{y}{\delta}\right)\right] dy$$

CAMBIO DI VAR $\quad Y = \frac{y}{\delta} \quad dY = \frac{1}{\delta} dy$

$$\theta = \delta \int_0^1 f(Y)\left[1-f(Y)\, dY\right] = \delta C_1$$

CASO LINEARE: $C_1 = \int_0^1 Y(1-Y)\, dY = \Big|_0^1 Y^2 - Y^3 = \frac{1}{2} - \frac{1}{3} = \boxed{\frac{1}{6}}$



$$\tau = \rho\, U^2 \frac{\theta}{dx} = \rho\, U\, C_1 \frac{\delta}{dx}$$

DALLA LEGGE DI NEWTON

$$\tau = \mu \frac{\partial u}{\partial y} \overset{\text{ADIMENSIONALIZZO}}{=} \mu\, U \frac{\partial}{\partial y}\left(\frac{u}{U}\right) \overset{\frac{1}{\delta}}{\frac{dY}{dy}}$$

$$= \mu\, U \underbrace{\frac{\partial f(Y)}{\partial Y}}_{C_2} \frac{1}{\delta} = \mu\, U\, C_2 \frac{1}{\delta}$$

$$\Rightarrow \rho \cancel{U} C_1 \frac{\delta}{dx} = \mu \cancel{U} \frac{C_2}{\delta} \rightarrow \int_0^{\delta_x} \delta\, d\delta = \mu \frac{C_2/C_1}{\rho\, U} \int_0^{dx} dx \rightarrow \frac{\delta^2}{2} = \mu \frac{C_2}{C_1 \rho\, U} x$$

$$\delta = \sqrt{2\mu \frac{C_2}{C_1 U} x} \rightarrow \text{ADIMENSIONALIZZO} \quad \frac{\delta}{x} = \sqrt{\frac{2\mu\, C_2/C_1\, x}{\rho\, U\, x^2}} = \sqrt{\frac{2\, C_2/C_1}{Re}}$$

↳ Lo spessore di strato limite per qualsiasi è inv prop a Re

$$|\vec{D}| = \rho\, b\, U^2 x \frac{\sqrt{2 C_2 C_1}}{\sqrt{Re}}$$

$$C_D = \frac{|\vec{D}|}{\frac{1}{2}\rho\, U \underbrace{b x}_{\text{Area}}} = \frac{\rho\, b\, U^2 x}{\frac{1}{2}\rho\, U^2 b\, x} \cdot \frac{\sqrt{2C_2C_1}}{\sqrt{Re_x}} = 2\frac{\sqrt{2C_2C_1}}{\sqrt{Re}} \rightarrow \text{All'aumentare di } Re \quad D\uparrow \quad C_D\downarrow$$